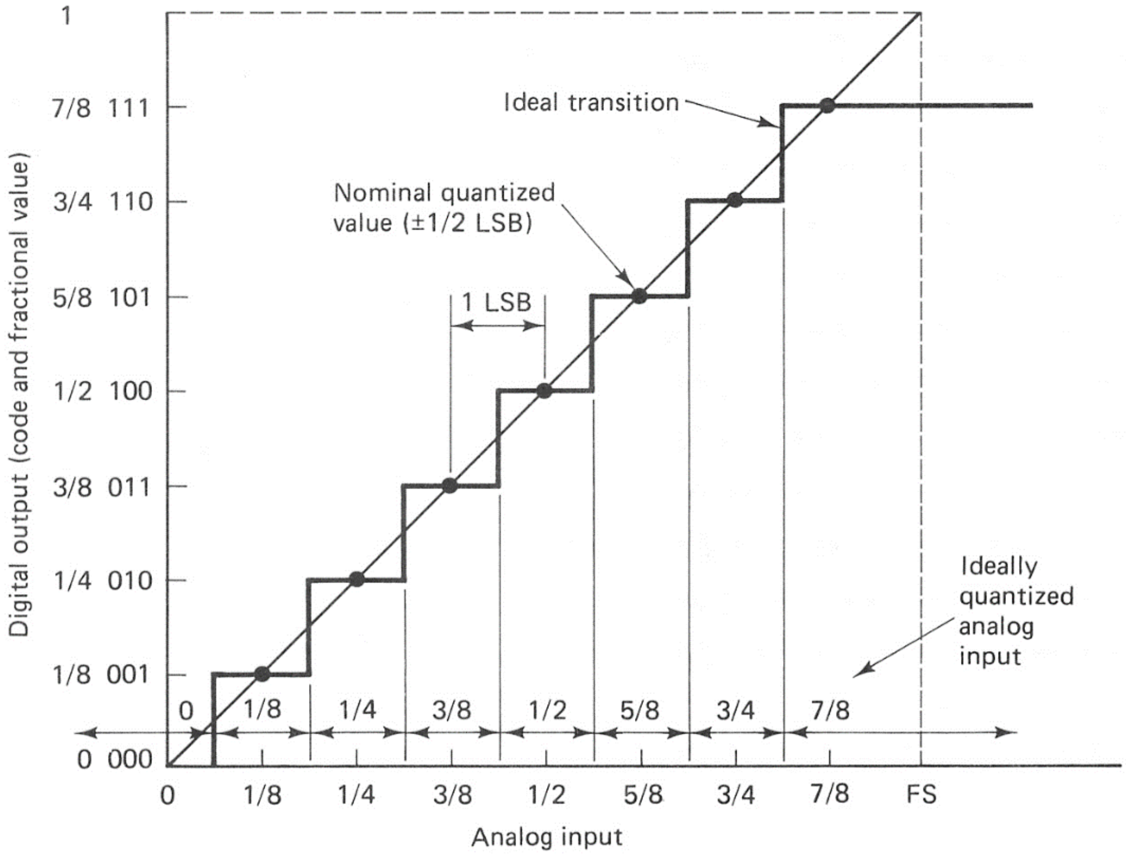
Analog/Digital Signal Interfacing

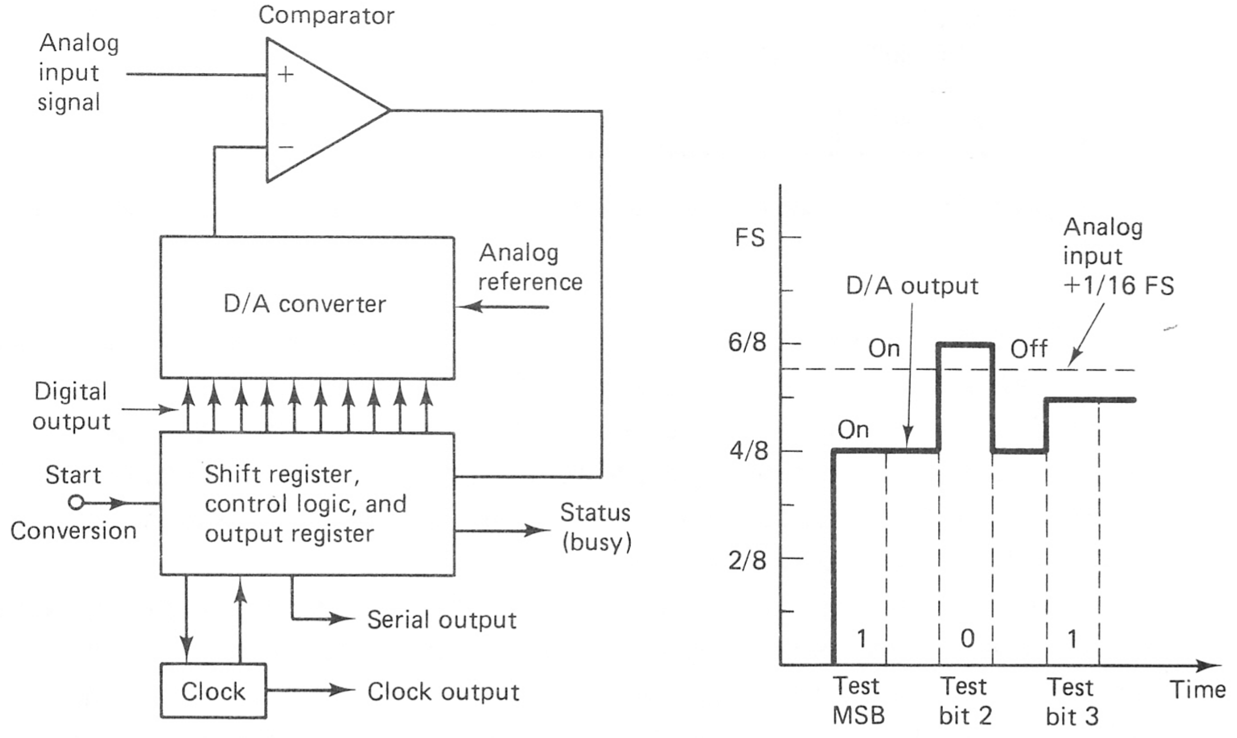
Implementation of Digital Filter using Cortex-M Microcontroller

Analog and Digital Signal

- Analog signal: Voltages, 1V, 3V, 10V,…
- Digital Signal: Binary numbers, 0100110011,….



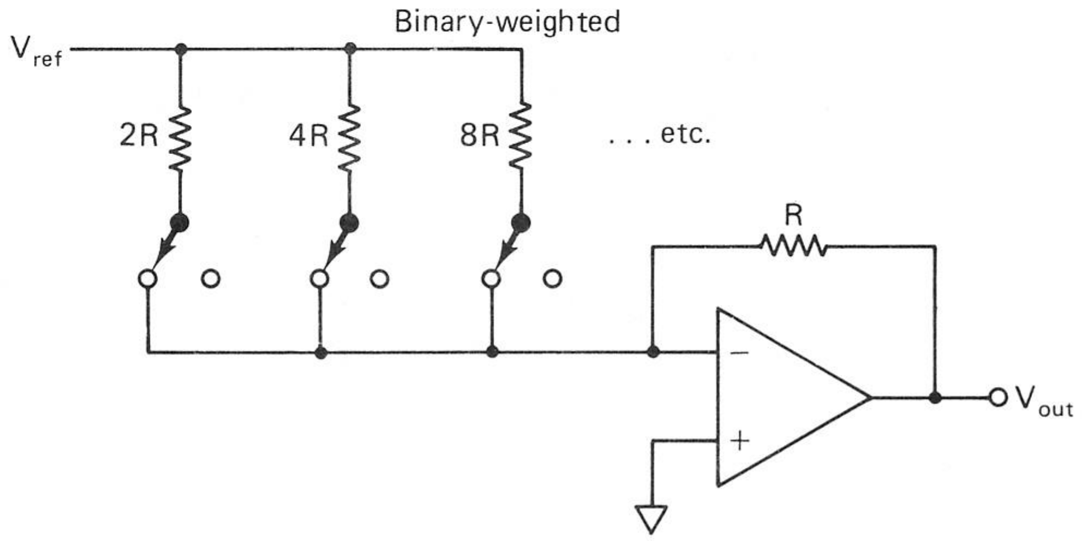
AD Converter



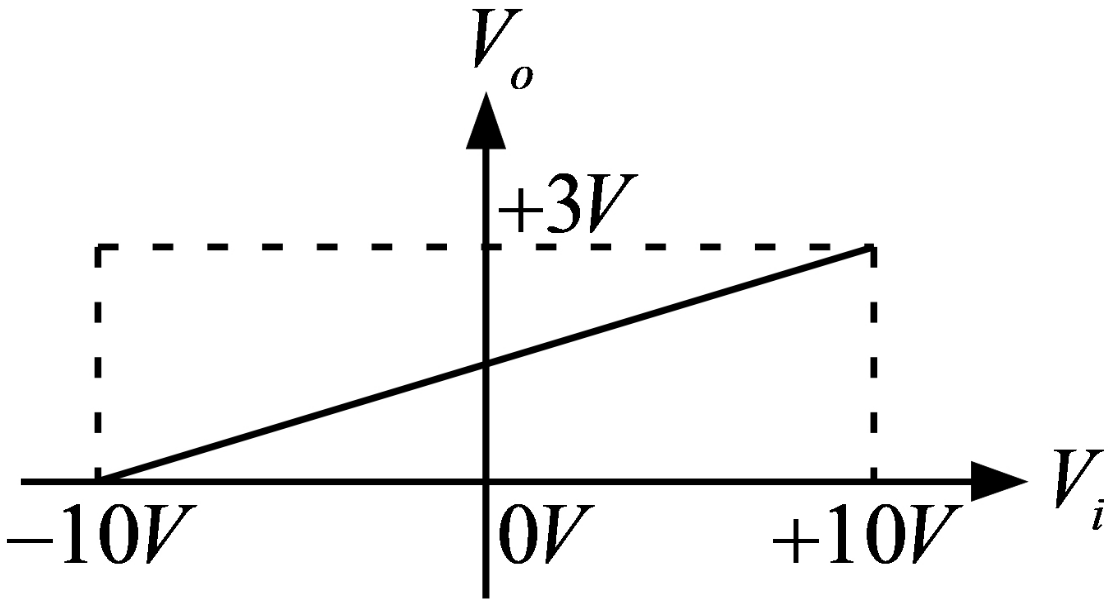
(a) (b)

DA Converter

Disadvantage:
Wide range of resistor values required



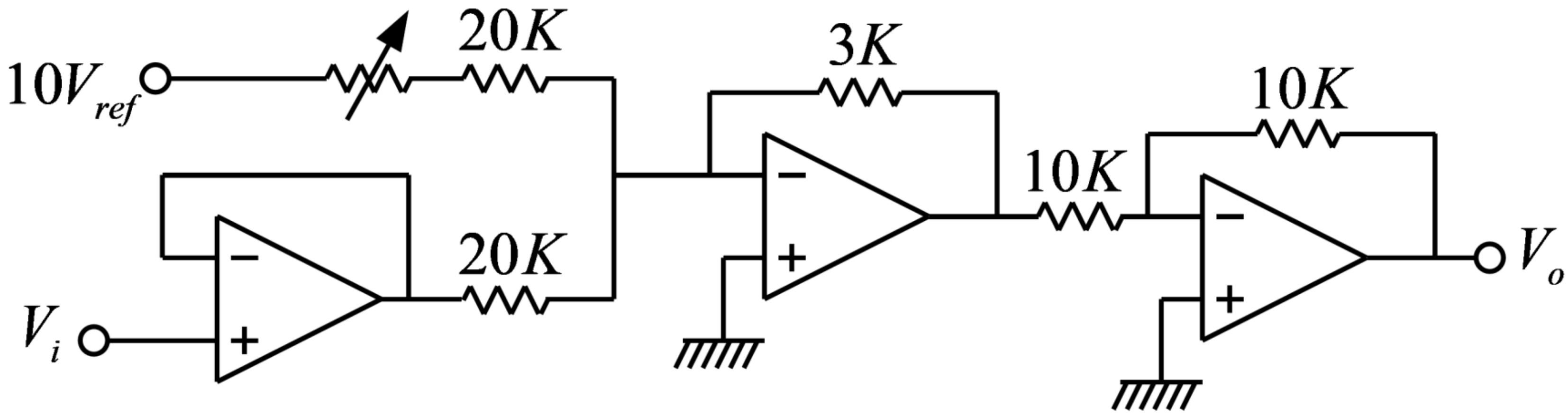
-10V~+10V to 0V~3V



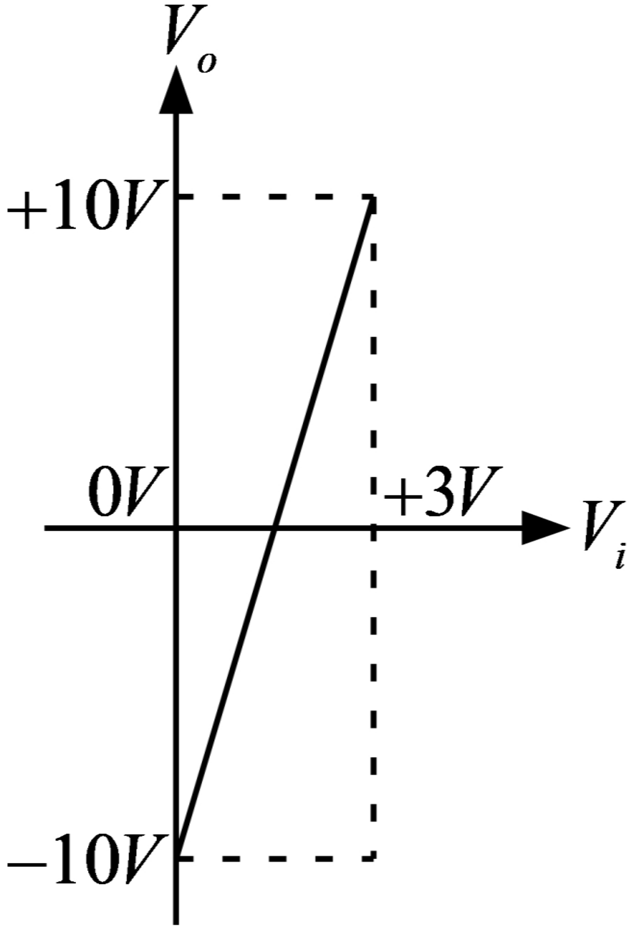
$$V_o = \frac{3}{20} V_i + \frac{3}{2}$$

-10V~+10V to 0V~3V

$$V_o = \frac{3}{20} V_i + \frac{3}{2}$$



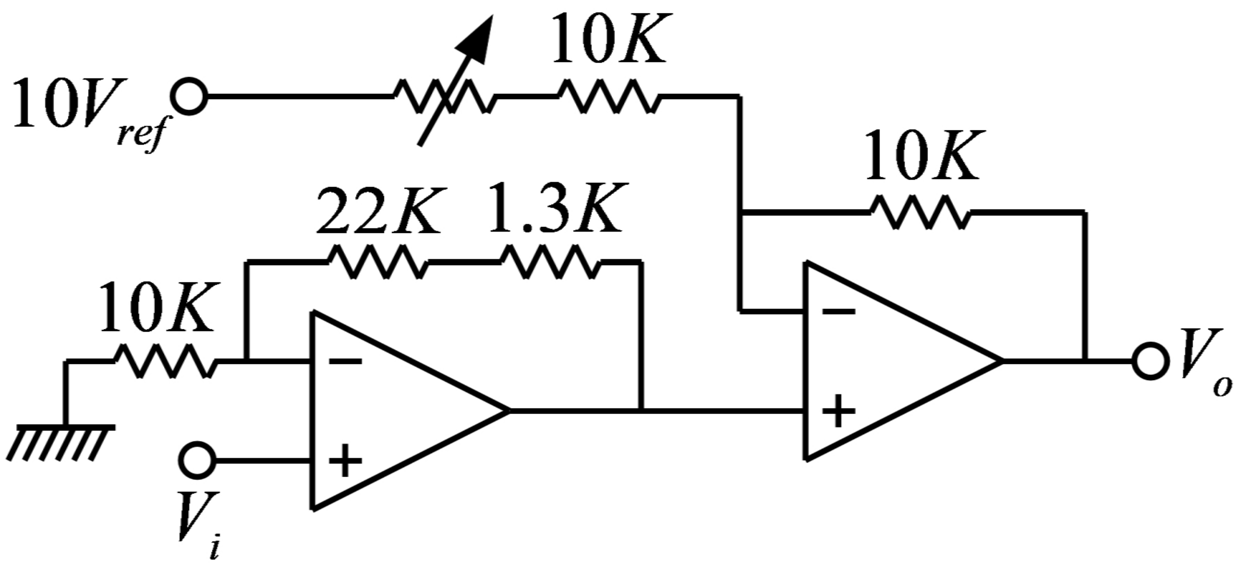
0V~3V to -10V~+10V



$$V_o = \frac{20}{3} V_i - 10$$

0V~3V to -10V~+10V

$$V_o = \frac{20}{3} V_i - 10$$

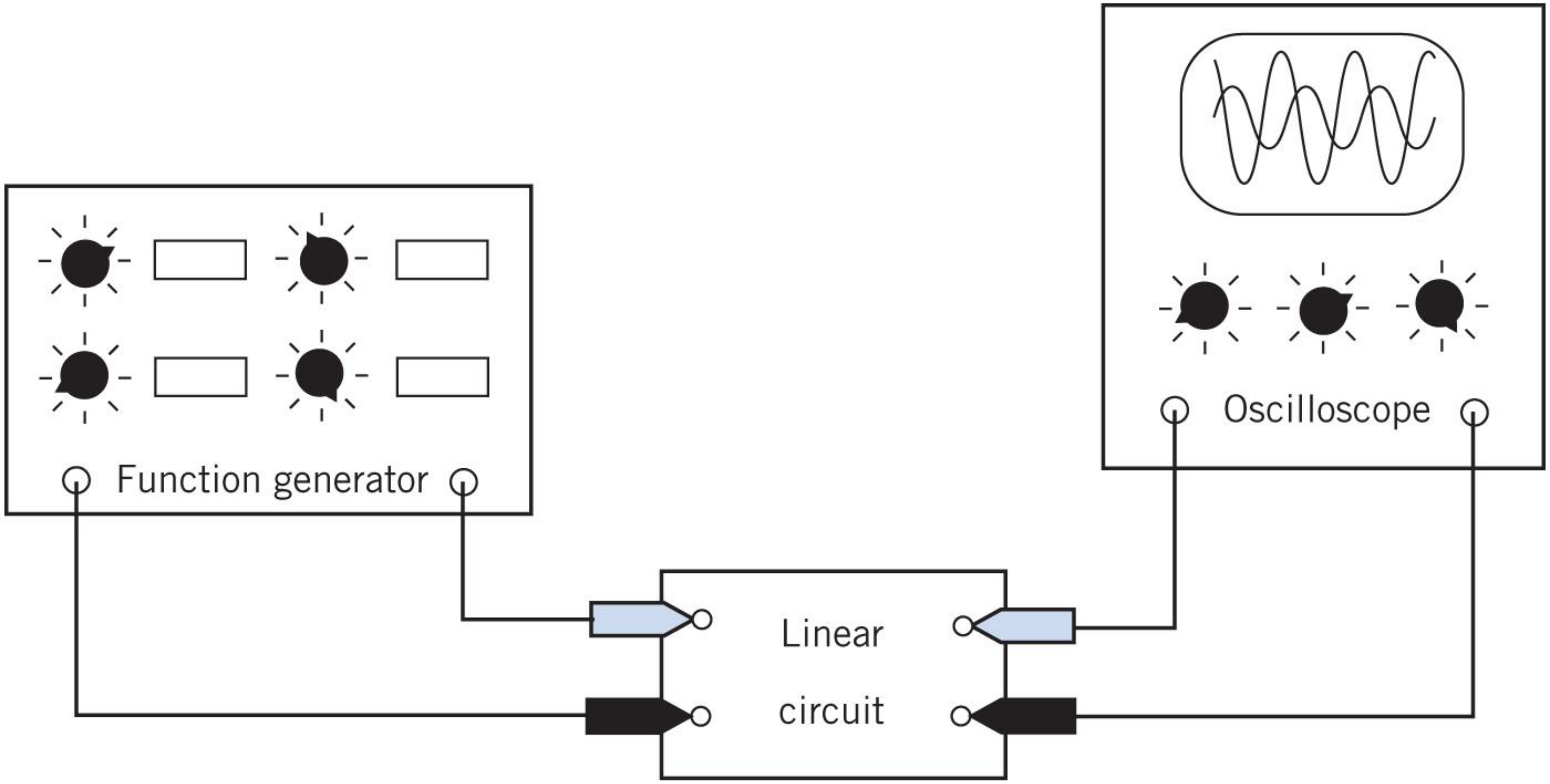


Two's Complement

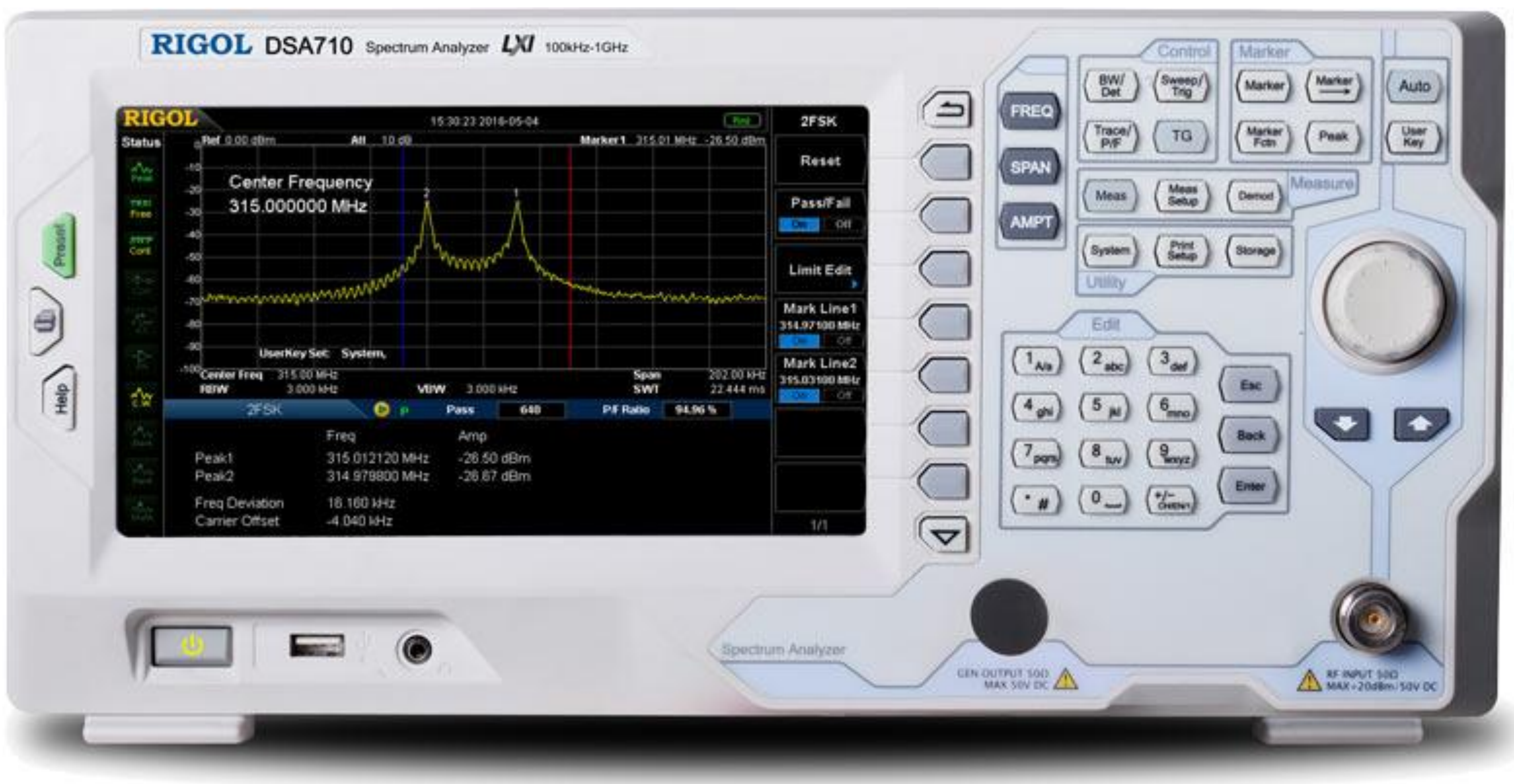
Input voltage(V)	A/D input (hexadecimal)	(decimal)	2's complement (hexadecimal)	(decimal)
(2047/2048)*10	0xFFF	4095	0x7FF	2047
...	...	...	...	...
(1/2048)*10	0x801	2049	0x001	1
0	0x800	2048	0x000	0
-(1/2048)*10	0x7FF	2047	0xFFF	-1
...	...	...	...	...
-10	0x000	0	0x800	-2048

Review: Frequency Response

- Define input and output
- Time response and frequency response

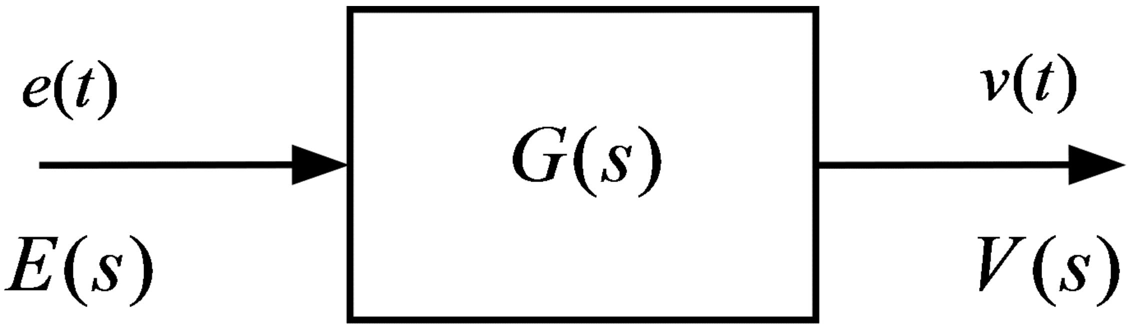


Spectrum Analyzer



Frequency Response

- Sinusoidal Input, Steady State

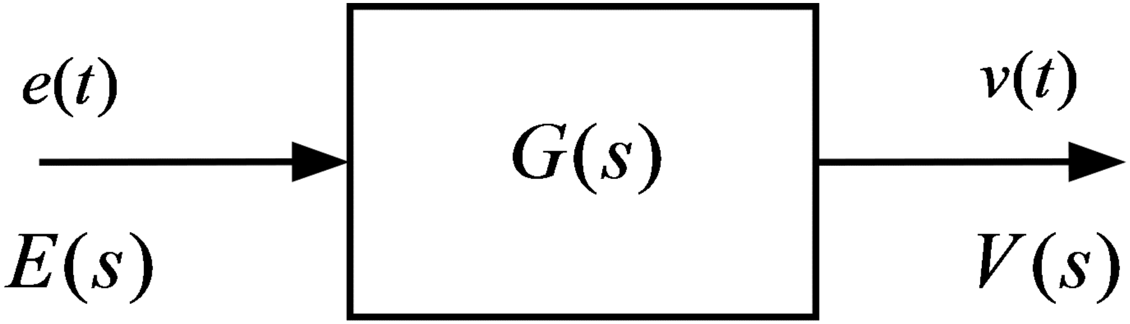


$$e(t) = A\cos(\omega t)$$

$$v(t) = A\left[\qquad\right]\cos\left(\omega t + \left[\qquad\right]\right)$$

Frequency Response

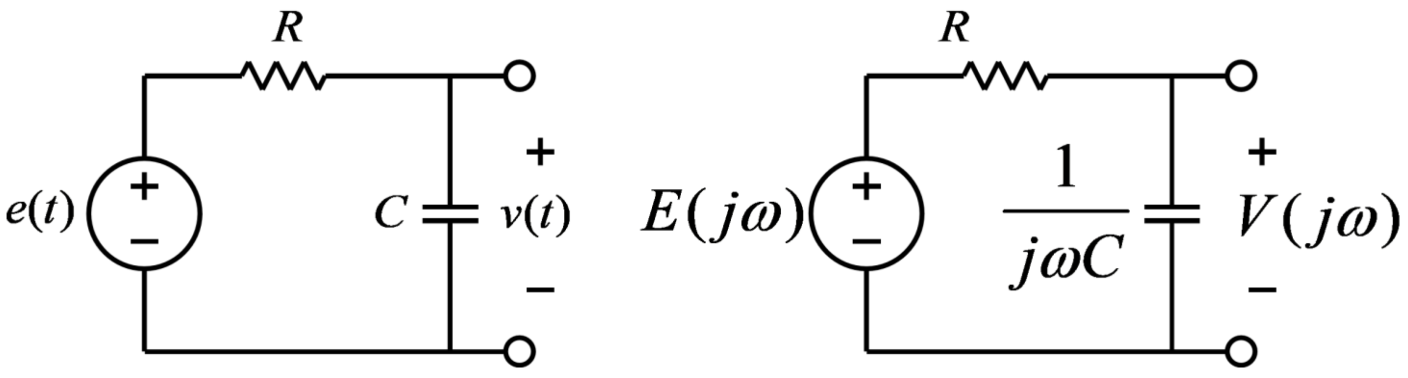
- Sinusoidal Input, Steady State



$$e(t) = A\cos(\omega t)$$

$$v(t) = A\left|G(j\omega)\right|\cos\left(\omega t + \measuredangle G(j\omega)\right)$$

Low Pass RC Circuit

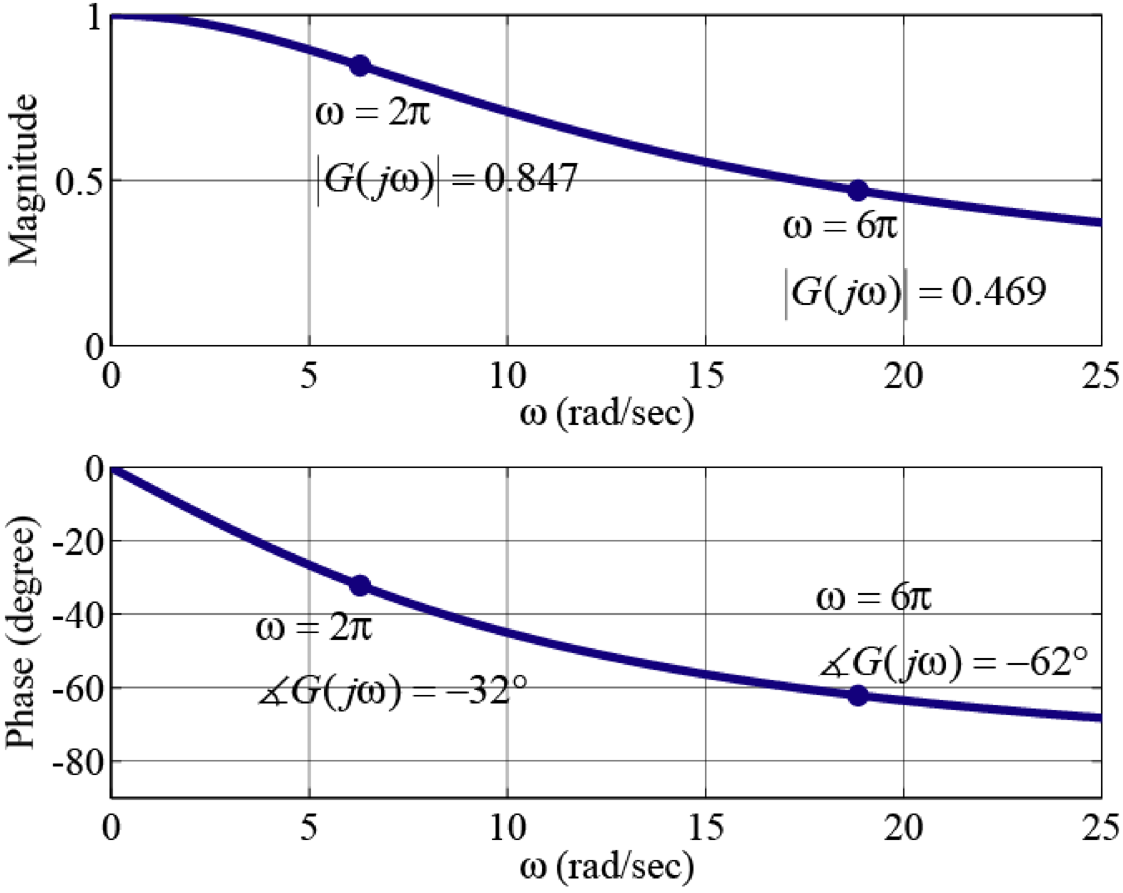


$$G(j\omega) = \frac{V(j\omega)}{E(j\omega)} = \frac{1}{j\omega RC + 1}$$

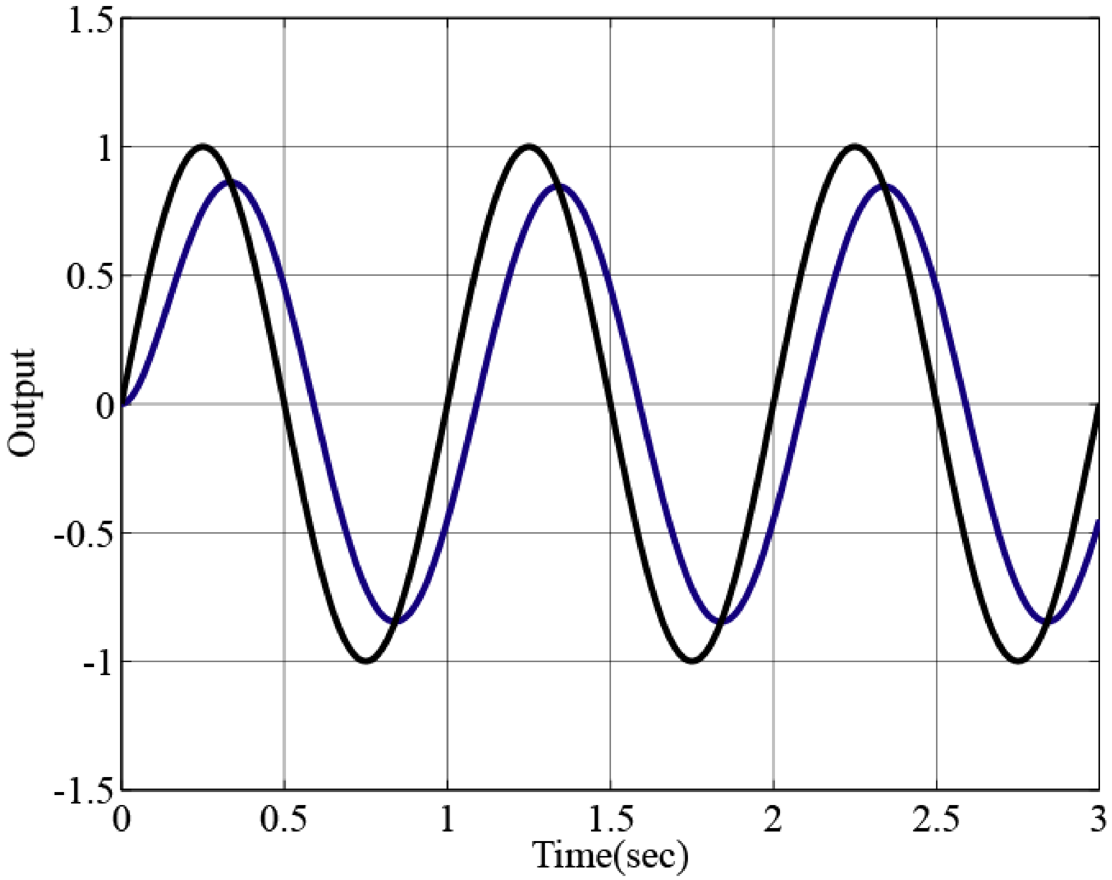
$$\left|G(j\omega)\right| = \left|\frac{1}{j\omega RC + 1}\right| = \frac{1}{\sqrt{(\omega RC)^2 + 1}}$$

$$\measuredangle G(j\omega) = \measuredangle\left(\frac{1}{j\omega RC + 1}\right) = -\tan^{-1}(\omega RC)$$

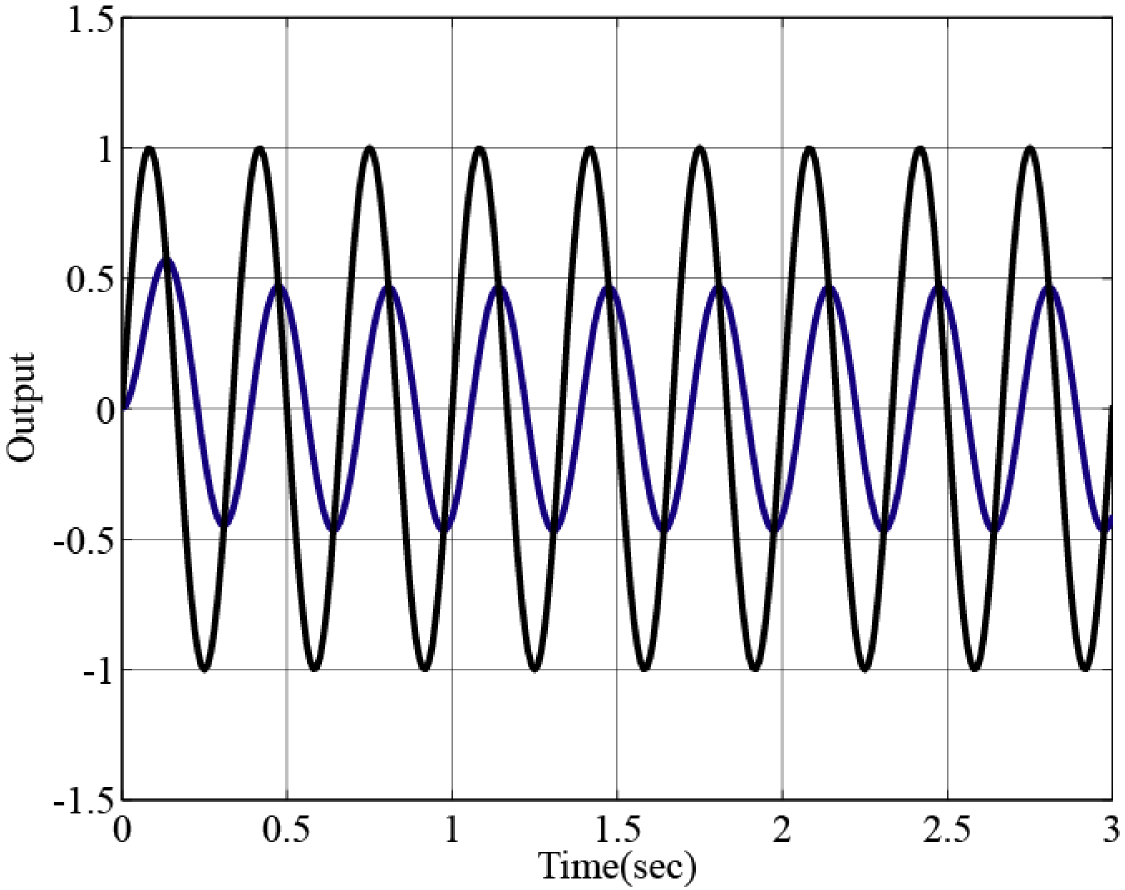
$$R = 100\Omega, C = 0.001F$$



1Hz

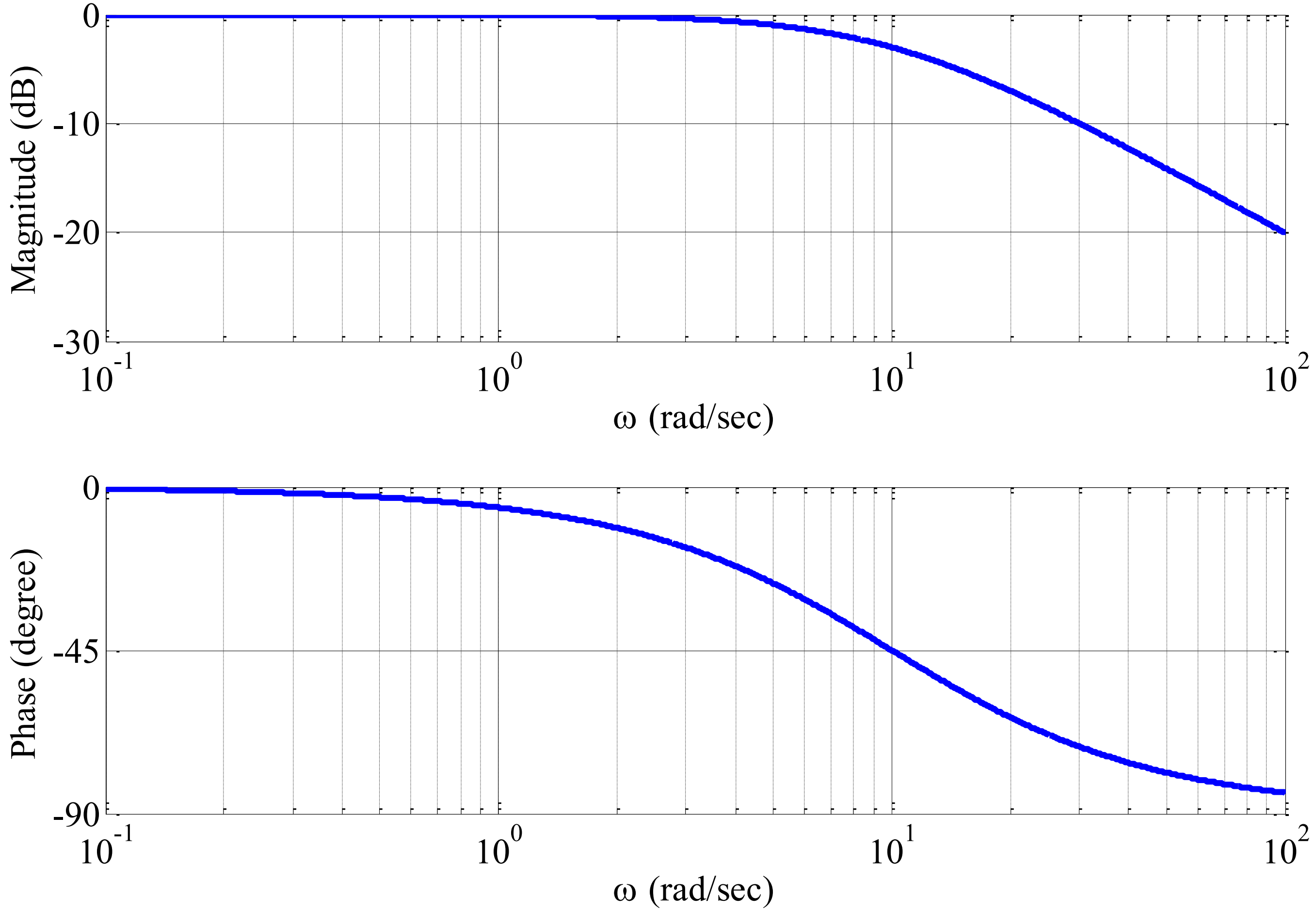


3Hz



Bode Plot

- dB(deci Bell): 20log(magnitude)



Band Width, Cut Off Frequency

$$G(j\omega)=\frac{V(j\omega)}{E(j\omega)}=\frac{1}{j\omega RC+1}$$

$$\left|G(j\omega)\right|=\left|\frac{1}{j\omega RC+1}\right|=\frac{1}{\sqrt{\left(\omega RC\right)^{2}+1}}$$

$$\omega RC=1\Rightarrow\left|G(j\omega)\right|=\left|\frac{1}{j+1}\right|=\frac{1}{\sqrt{2}}\Rightarrow 20\log_{10}\frac{1}{\sqrt{2}}=-3dB$$

$$\omega=\frac{1}{RC}$$

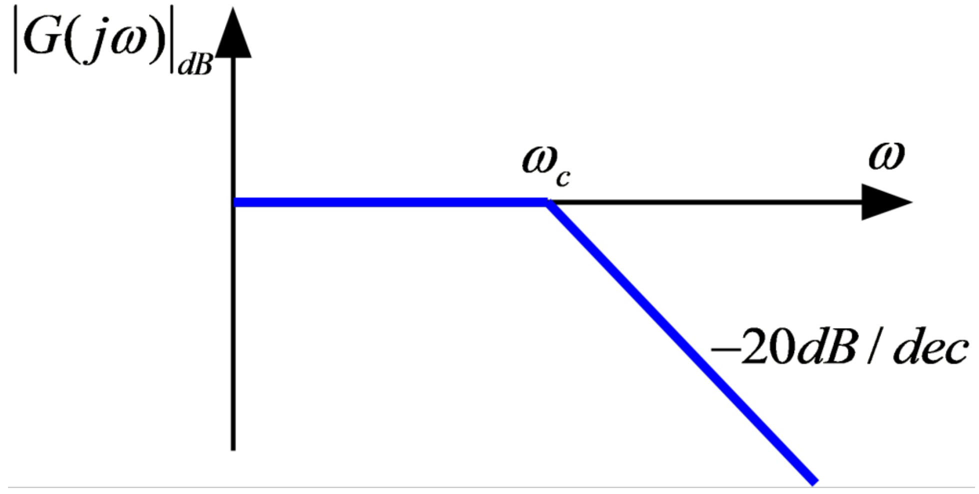
1st Order Digital Filter

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{1 + s/\omega_c} = \frac{\omega_c}{\omega_c + s}$$

$$sY(s) + \omega_c Y(s) = \omega_c U(s)$$

$$\dot{y}(t) + \omega_c y(t) = \omega_c u(t)$$

$$\dot{y}(t) \approx \frac{y(t) - y(t - \Delta t)}{\Delta t}$$



Digital Approximation

$$\frac{y(t)-y(t-\Delta t)}{\Delta t}+\omega_c y(t)=\omega_c u(t)$$

$$y(t)-y(t-\Delta t)+\Delta t\omega_c y(t)=\Delta t\omega_c u(t)$$

$$\left(1+\Delta t\omega_c\right)y(t)=y(t-\Delta t)+\Delta t\omega_c u(t)$$

$$y(t)=\frac{1}{1+\Delta t\omega_c}\left(y(t-\Delta t)+\Delta t\omega_c u(t)\right)$$

2nd Order Digital Filter

$$\frac{Y(s)}{U(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$s^2 Y(s) + 2\zeta\omega_n s Y(s) + \omega_n^2 Y(s) = \omega_n^2 U(s)$$

$$\frac{d^2 y}{dt^2} + 2\zeta\omega_n \frac{dy}{dy} + \omega_n^2 y = \omega_n^2 u$$

Review: 2차 시스템의 과도 응답

- 표준형 2차 시스템

$$\frac{Y(s)}{R(s)} = G_T(s) = \frac{1}{\left(s/\omega_n\right)^2 + \left(2\zeta s/\omega_n\right) + 1} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- 단위 계단 응답

$$U_s(s) = 1/s$$

- 특성 방정식

$$Y(s) = G_T(s)U_s(s) = \frac{\omega_n^2}{\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)s}$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

두 개의 서로 다른 근을 가지는 경우의 해(solution)

$$Y(s) = \frac{\omega_n^2}{(s-s_1)(s-s_2)s} = \frac{K_1}{s-s_1} + \frac{K_2}{s-s_2} + \frac{1}{s}$$

$$K_1 = \frac{\omega_n^2}{(s_1-s_2)s_1} \qquad K_2 = \frac{\omega_n^2}{(s_2-s_1)s_2}$$

$$y(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t} + 1, t \geq 0$$

2중근을 가지는 경우의 해(solution)

$$Y(s)=\frac{\omega_n^2}{(s-s_1)^2 s}=\frac{K_1}{s-s_1}+\frac{K_2}{(s-s_1)^2}+\frac{1}{s}$$

$$K_1=-\frac{\omega_n^2}{s_1^2} \qquad K_2=\frac{\omega_n^2}{s_1}$$

$$y(t)=K_1e^{s_1t}+K_2te^{s_1t}+1$$

Review: 2차 시스템의 특성 방정식

- 미분 방정식

$$\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)Y(s) = \omega_n^2 U_s(s)$$

$$\ddot{y}(t) + 2\zeta\omega_n \dot{y}(t) + \omega_n^2 y(t) = \omega_n^2 u_s(t)$$

- 특성 방정식

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = \left(s - s_1\right)\left(s - s_2\right) = 0$$

- 특성 방정식의 근

$$s_1 = -\zeta\omega_n + \sqrt{\left(\zeta^2 - 1\right)\omega_n^2} \qquad s_2 = -\zeta\omega_n - \sqrt{\left(\zeta^2 - 1\right)\omega_n^2}$$

특성 방정식이 2개의 실수 근을 가지는 경우: $\zeta > 1$

- 과잉 감쇄(over-damped)

$$\tau_1 = -s_1 = \zeta\omega_n - \sqrt{\left(\zeta^2 - 1\right)\omega_n^2}$$

$$\tau_2 = -s_2 = \zeta\omega_n + \sqrt{\left(\zeta^2 - 1\right)\omega_n^2}$$

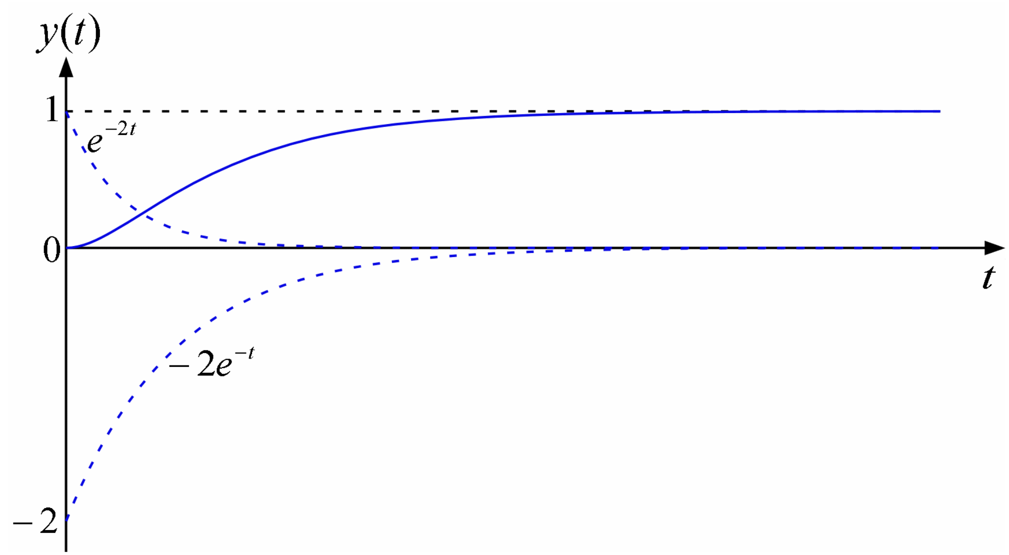
$$y(t) = K_1 e^{-\tau_1 t} + K_2 e^{-\tau_2 t} + 1, t \geq 0$$

과잉 감쇄 예제

$$G(s) = \frac{2}{s^2 + 3s + 2} = \frac{2}{(s+1)(s+2)}$$

$$Y(s) = \frac{2}{(s+1)(s+2)s} = -\frac{2}{s+1} + \frac{1}{s+2} + \frac{1}{s}$$

$$y(t) = -2e^{-t} + e^{-2t} + 1, t \geq 0$$



특성 방정식이 2개의 복소수 근을 가지는 경우: $0 < \zeta < 1$

- 부족 감쇄(under-damped)

$$s_1 = -\zeta\omega_n + \sqrt{\left(\zeta^2 - 1\right)\omega_n^2} = -\zeta\omega_n + j\sqrt{\left(1-\zeta^2\right)\omega_n^2}$$

$$s_2 = -\zeta\omega_n - \sqrt{\left(\zeta^2 - 1\right)\omega_n^2} = -\zeta\omega_n - j\sqrt{\left(1-\zeta^2\right)\omega_n^2}$$

$$\sigma = -\zeta\omega_n$$

$$\omega_d = \sqrt{\left(1-\zeta^2\right)\omega_n^2} = \omega_n\sqrt{1-\zeta^2}$$

$$s_1 = \sigma + j\omega_d$$

$$s_2 = \sigma - j\omega_d$$

특성 방정식이 2개의 복소수 근을 가지는 경우: $0 < \zeta < 1$

$$y(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t} + 1 = K_1 e^{s_1 t} + K_1^* e^{s_1^* t} + 1, t \ge 0$$

$$K_1 = \frac{\omega_n^2}{2 j\omega_d \left(\sigma + j\omega_d\right)}$$

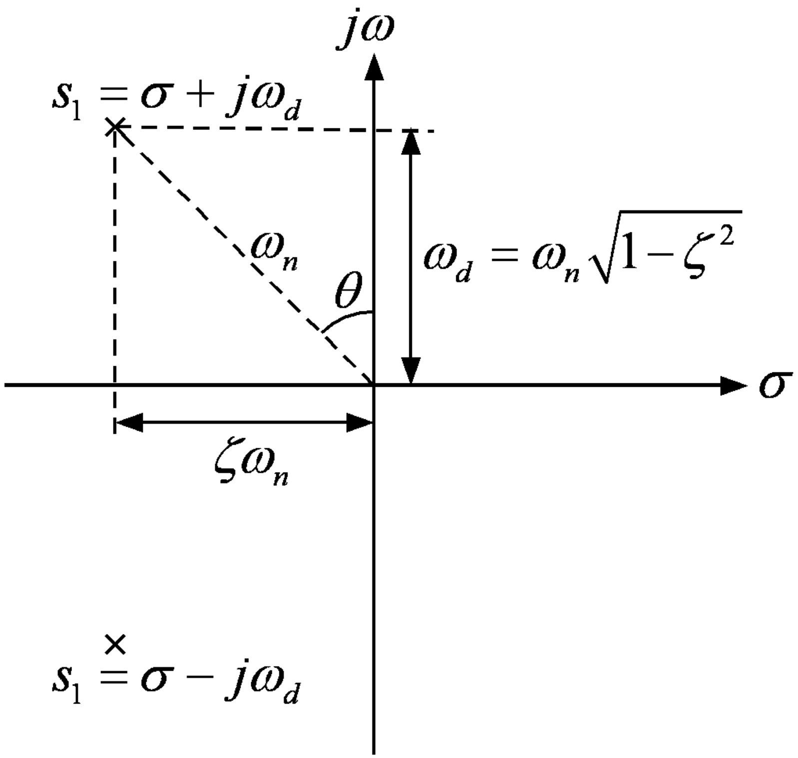
$$\begin{aligned}
y(t) &= K_1 e^{\sigma t + j\omega_d t} + K_1^* e^{\sigma t - j\omega_d t} + 1 \\
&= \left|K_1\right| e^{j\measuredangle K_1} e^{\sigma t + j\omega_d t} + \left|K_1\right| e^{-j\measuredangle K_1} e^{\sigma t - j\omega_d t} + 1 \\
&= \left|K_1\right| e^{\sigma t} \left(e^{j\measuredangle K_1} e^{j\omega_d t} + e^{-j\measuredangle K_1} e^{-j\omega_d t} \right) + 1 \\
&= 2\left|K_1\right| e^{\sigma t} \cos\left(\omega_d t + \measuredangle K_1\right) + 1, t \ge 0
\end{aligned}$$

특성 방정식이 2개의 복소수 근을 가지는 경우: $0 < \zeta < 1$

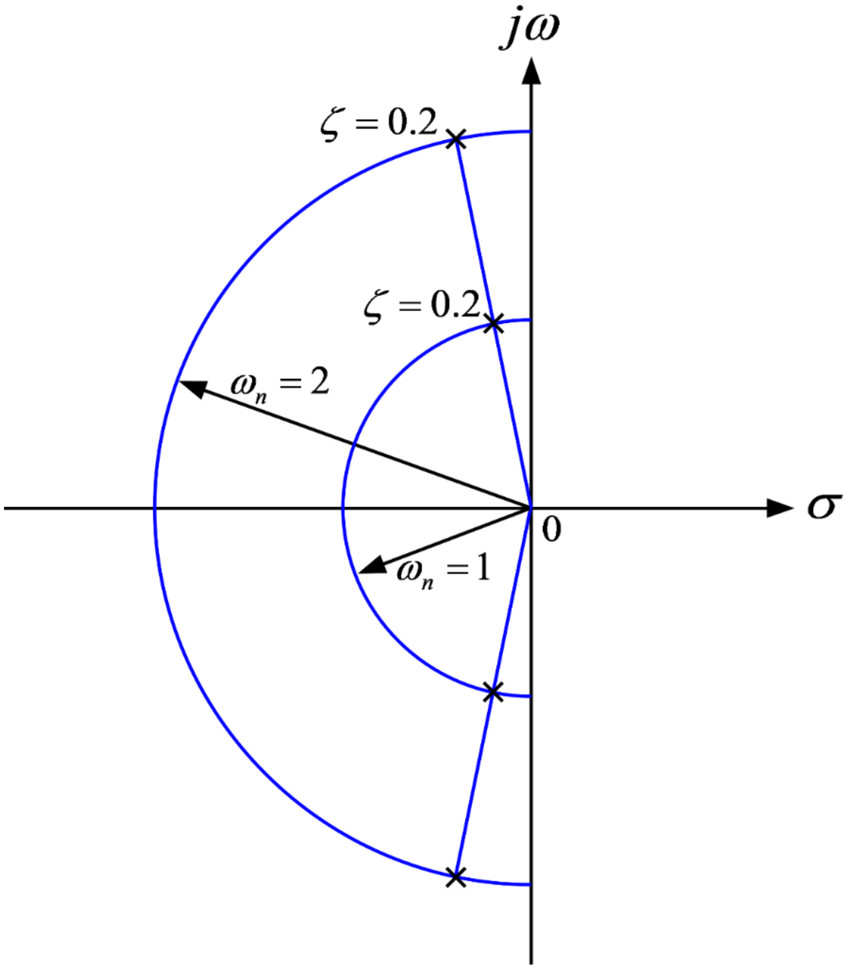
$$y(t) = 2|K_1|e^{-\zeta\omega_n t}\cos\left(\omega_n\sqrt{1-\zeta^2}t + \measuredangle K_1\right) + 1$$

$$= -\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\cos\left(\omega_n\sqrt{1-\zeta^2}t - \phi\right) + 1, t \geq 0$$

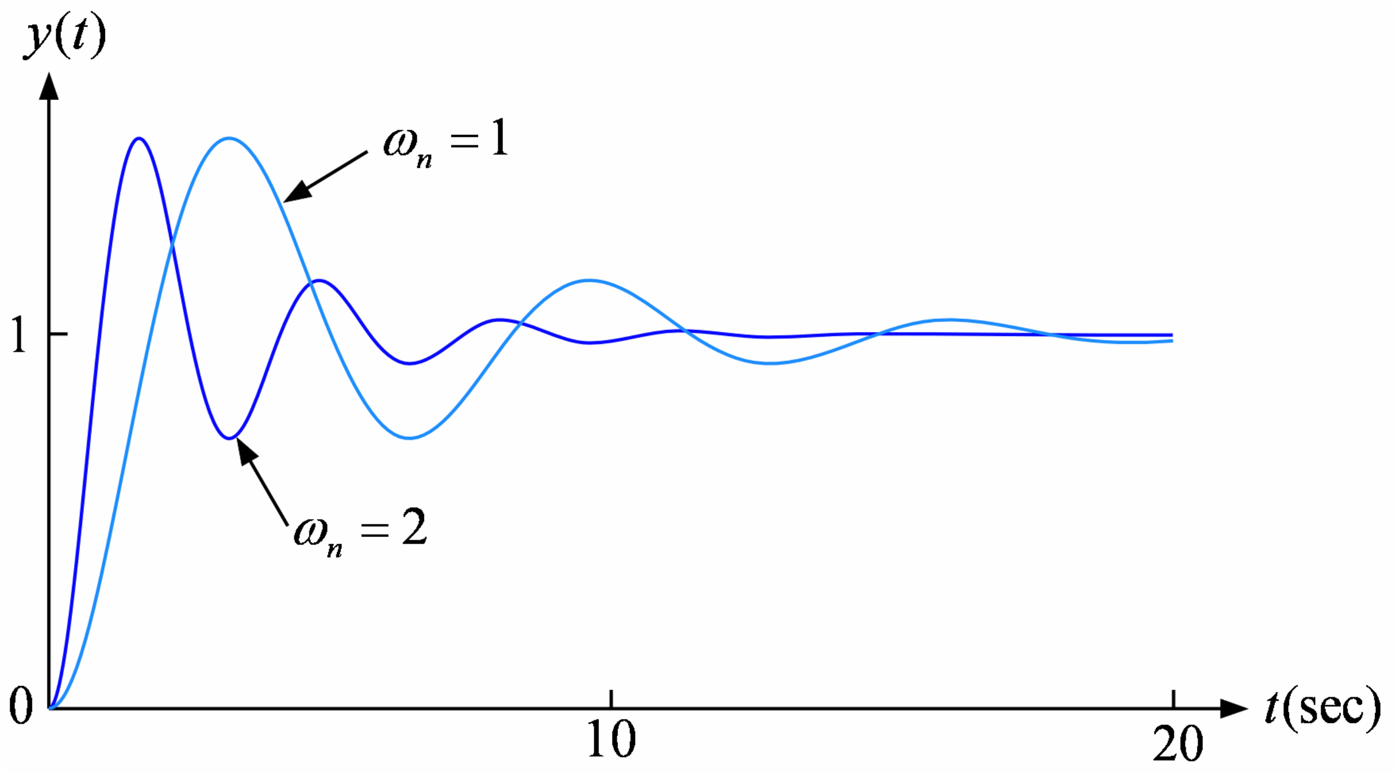
$$\phi = \tan^{-1}\frac{\zeta}{\sqrt{1-\varsigma^2}}$$



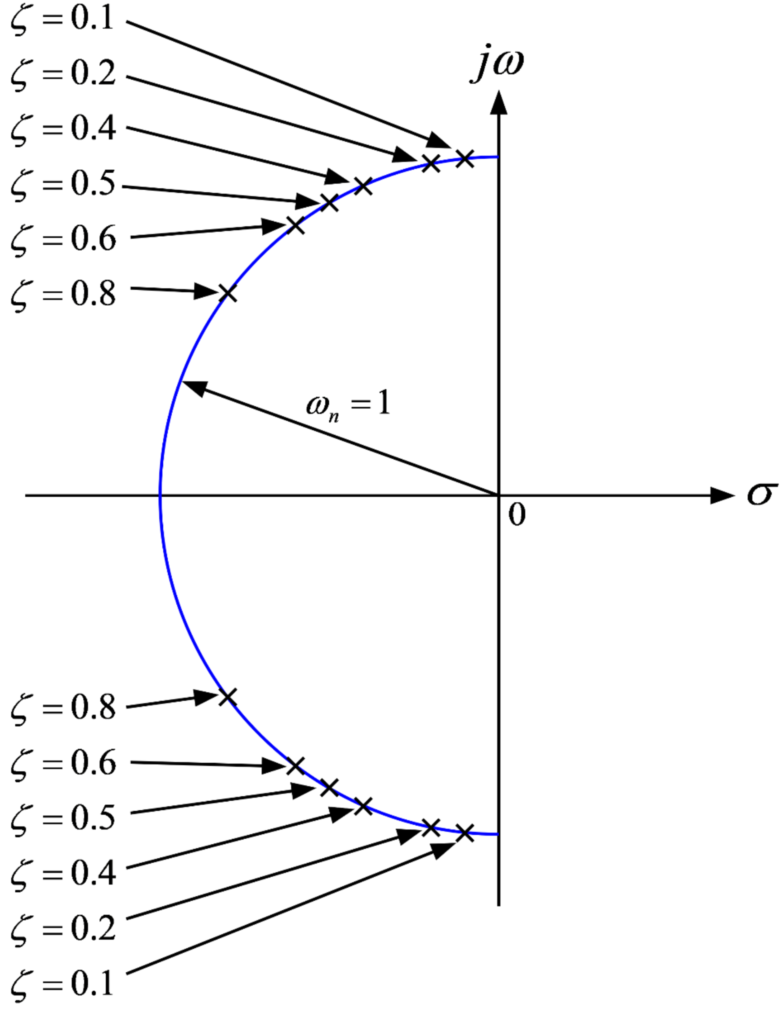
ω_n 의 변화에 따른 극점의 위치와 응답 속도의 변화



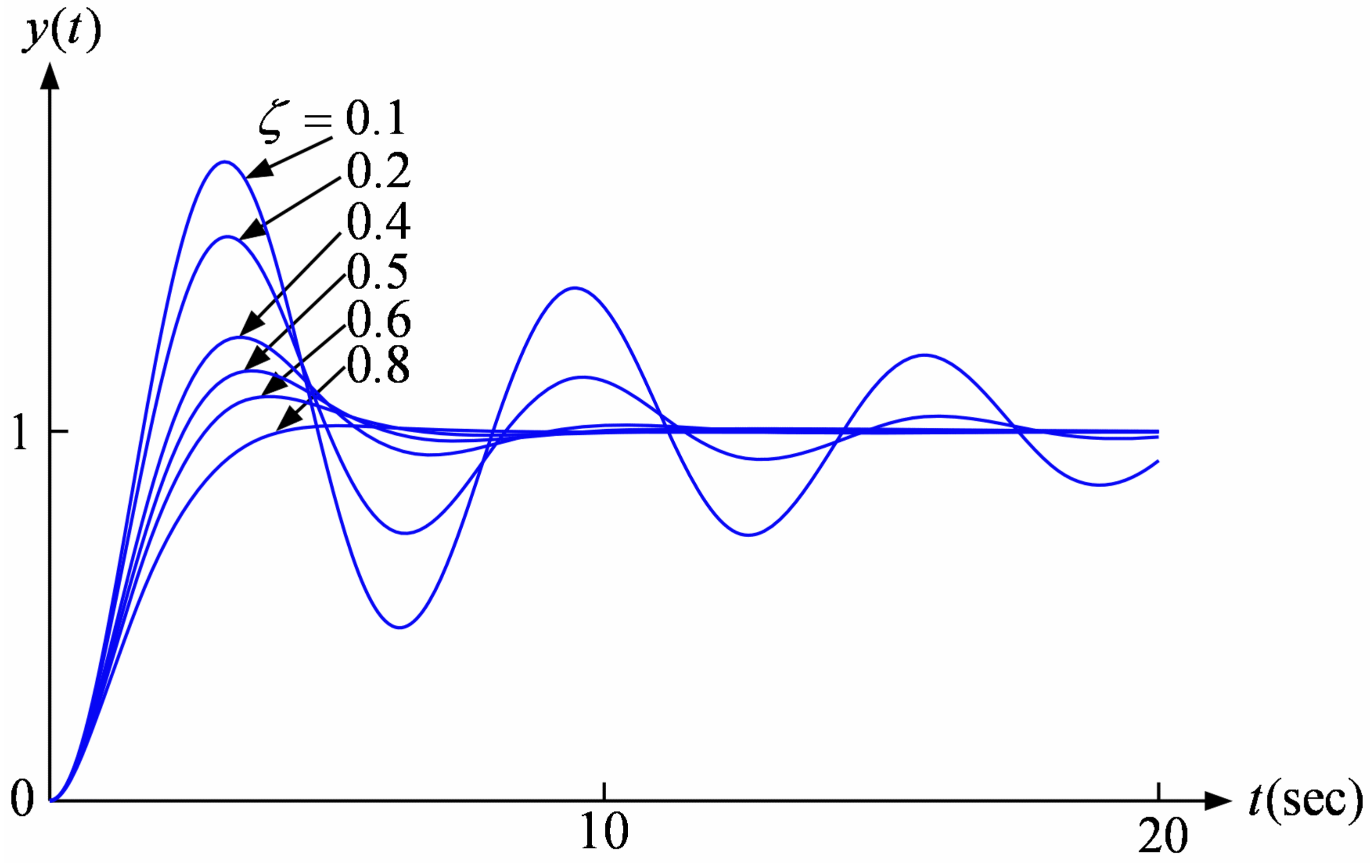
ω_n 의 변화에 따른 극점의 위치와 응답 속도의 변화



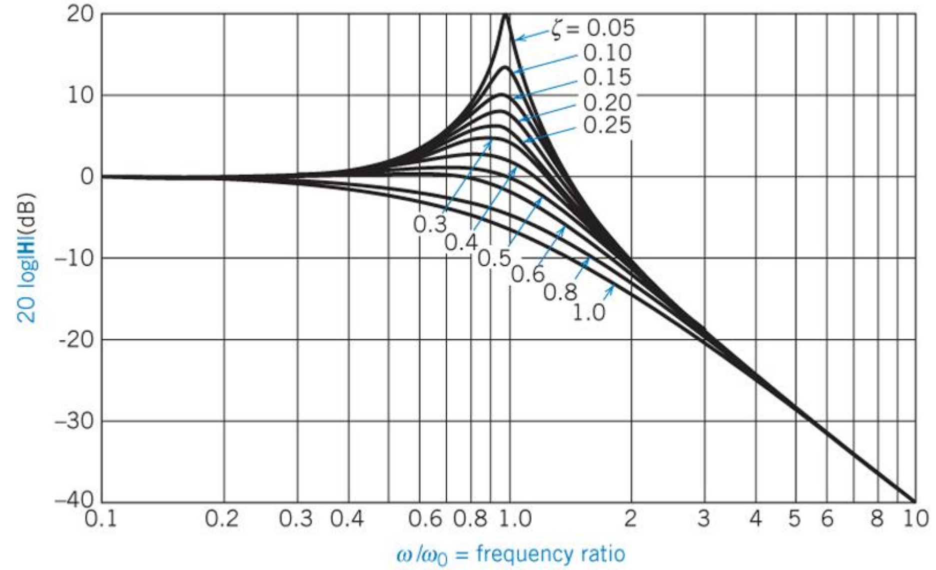
ζ의 변화에 따른 극점의 위치와 응답 속도의 변화



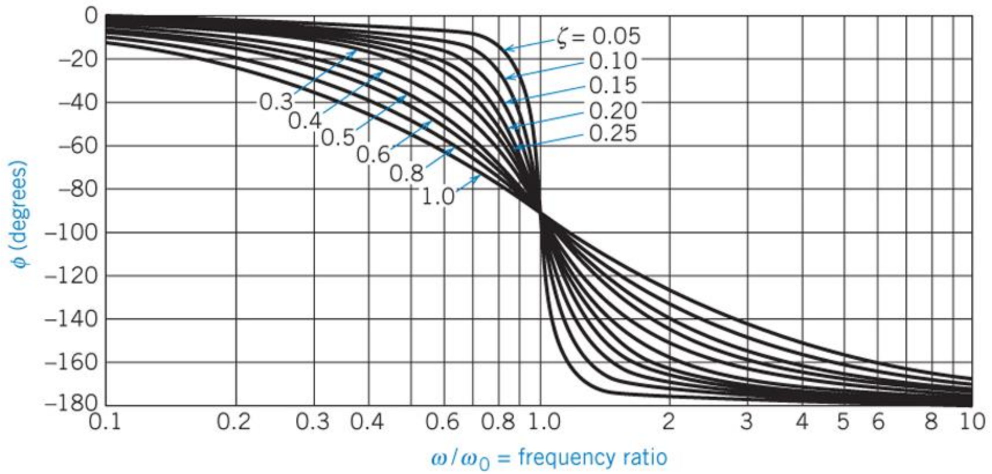
ζ의 변화에 따른 극점의 위치와 응답 속도의 변화



Frequency Response of 2nd Order System



(a)



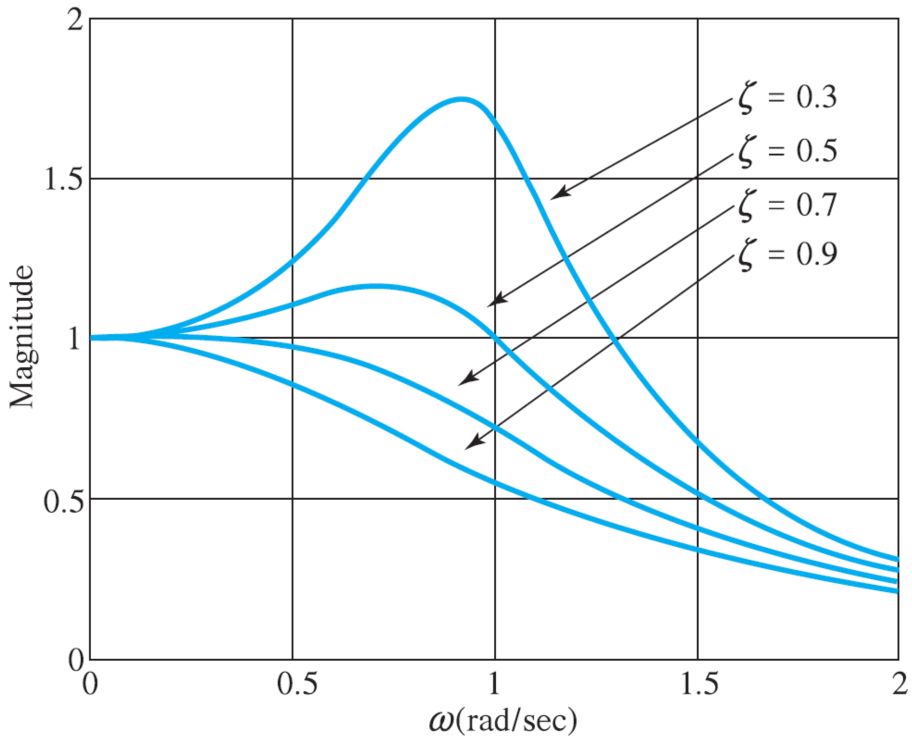
(b)

주파수 응답과 극점의 관계

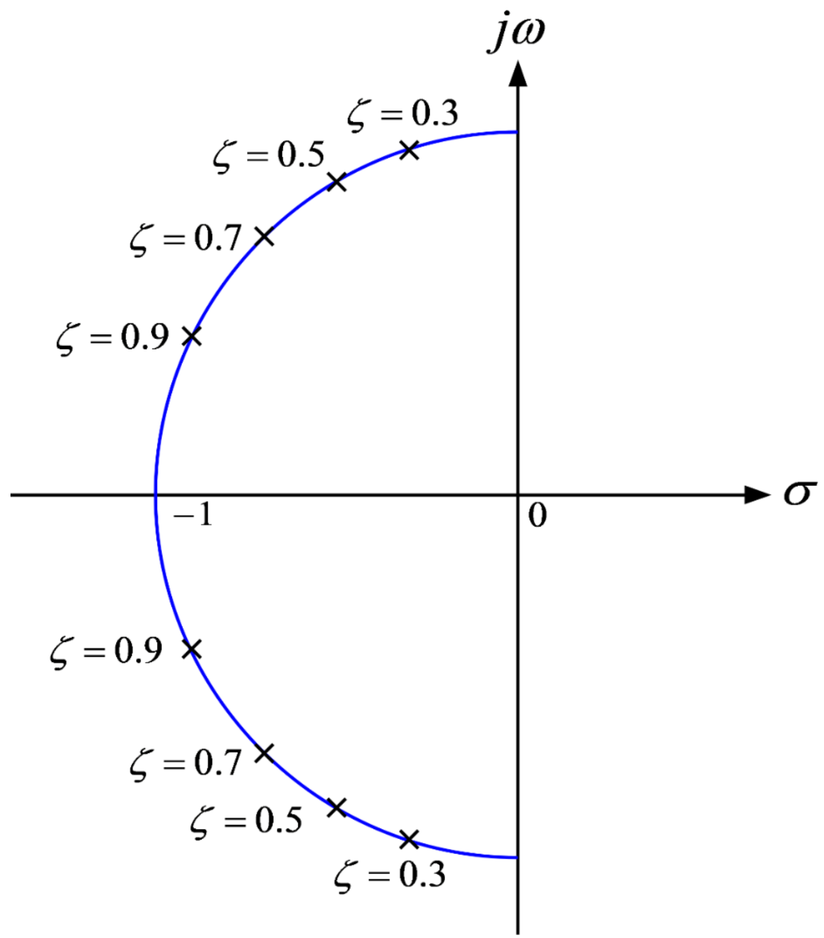
$$G(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}=\frac{1}{\left(s/\omega_n\right)^2+\left(2\zeta s/\omega_n\right)+1}$$

$$\left|G(s)\right|$$

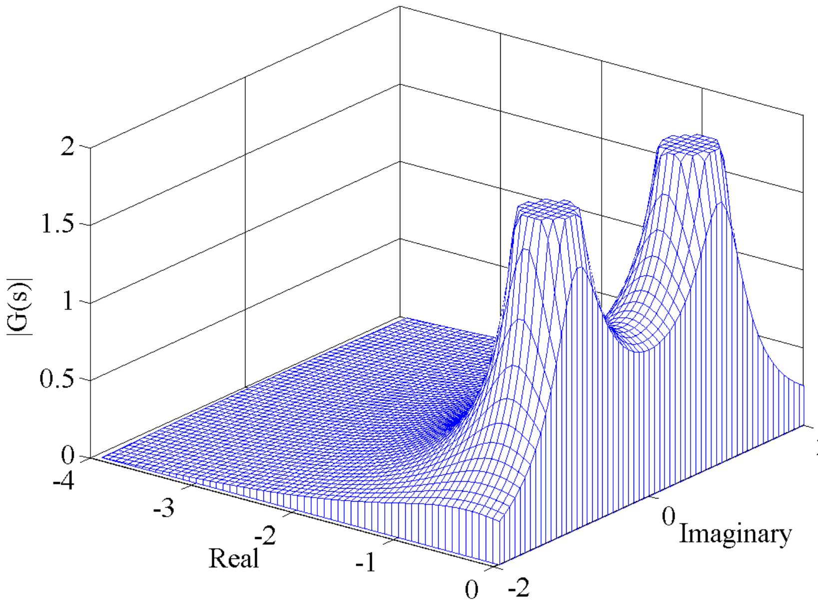
$$=\left|\frac{1}{\left(s/\omega_n\right)^2+\left(2\zeta s/\omega_n\right)+1}\right|$$



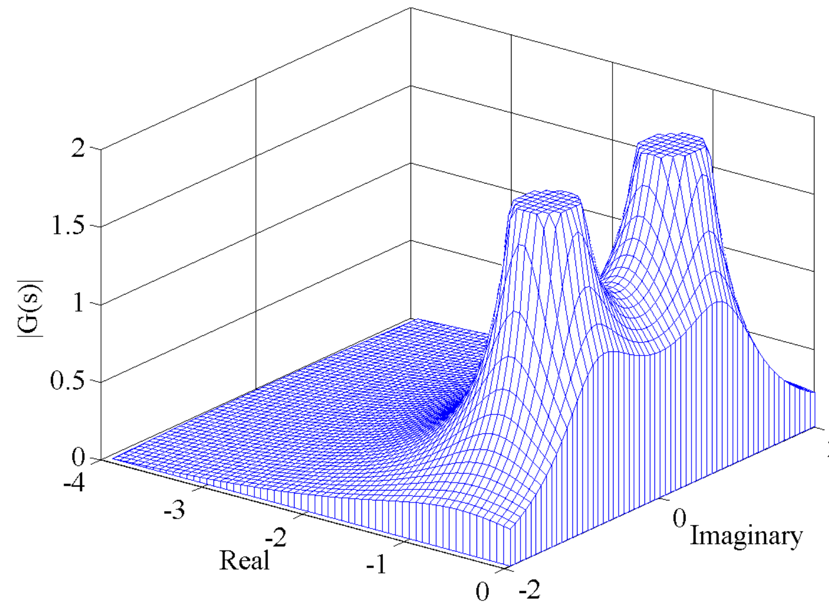
표준형 2차 시스템의 극점



표준형 2차 시스템의 $\left|G(s)\right|$

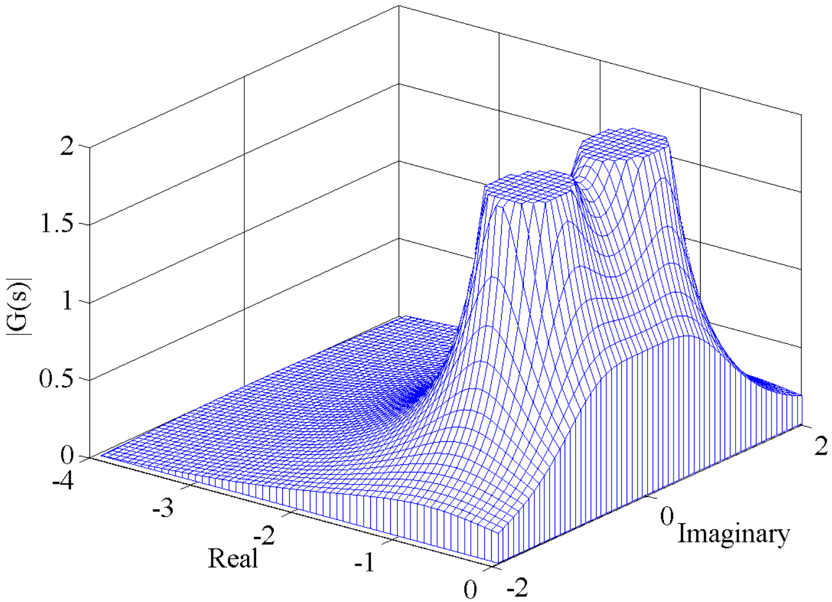


$\zeta = 0.3$

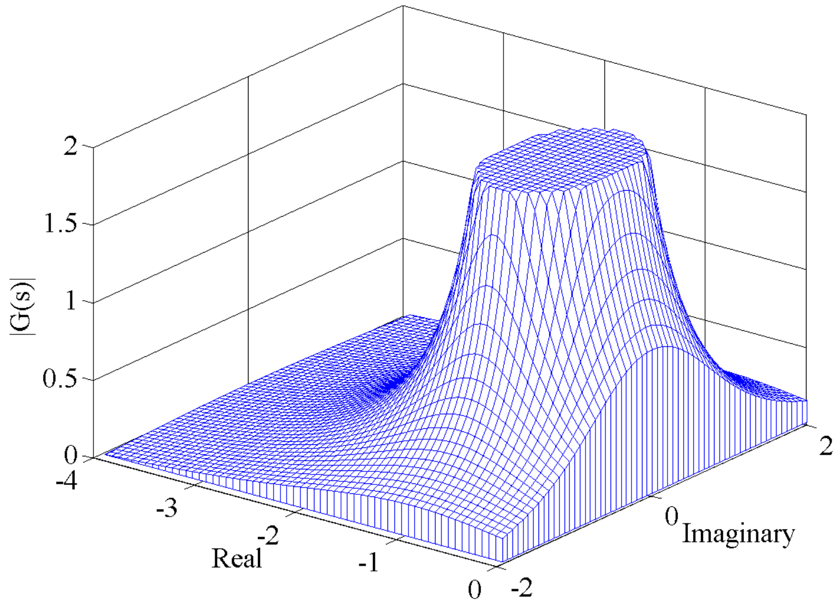


$\zeta = 0.5$

표준형 2차 시스템의 $|G(s)|$



$\zeta = 0.7$



$\zeta = 0.9$