Automation Programming

Nonlinear System

Magnetic Levitation

비선형 시스템의 선형화

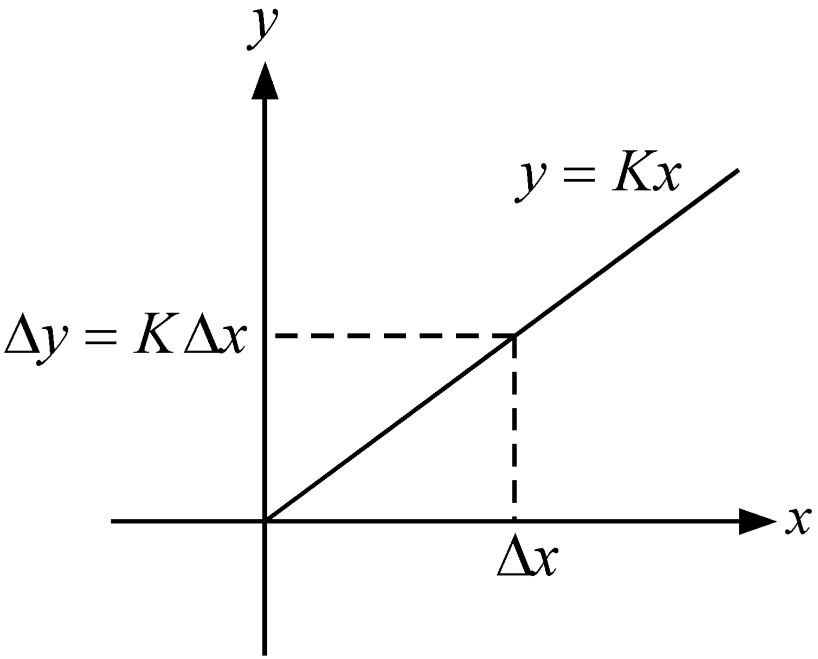
- 선형 함수의 정의

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

$$f(\alpha x) = \alpha f(x)$$

- 선형 함수

$$y = Kx$$



- 비선형 함수

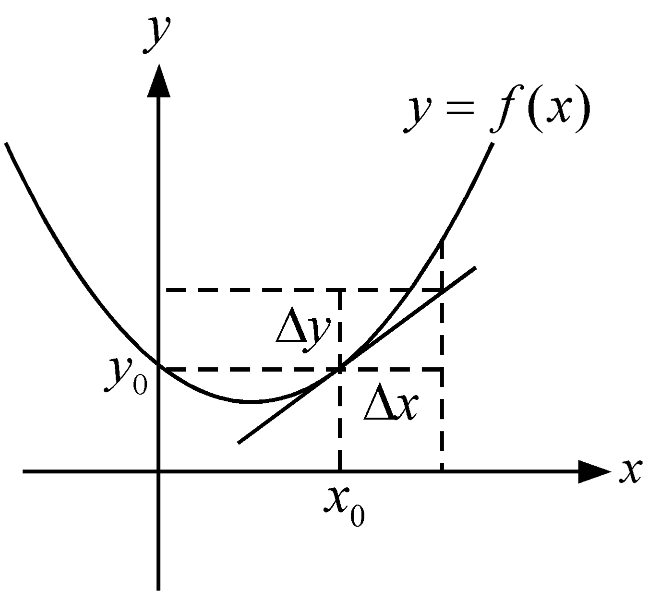
$$y = f(x)$$

- 선형화 중심점

$$y_0 = f\left(x_0\right)$$

- 근사식

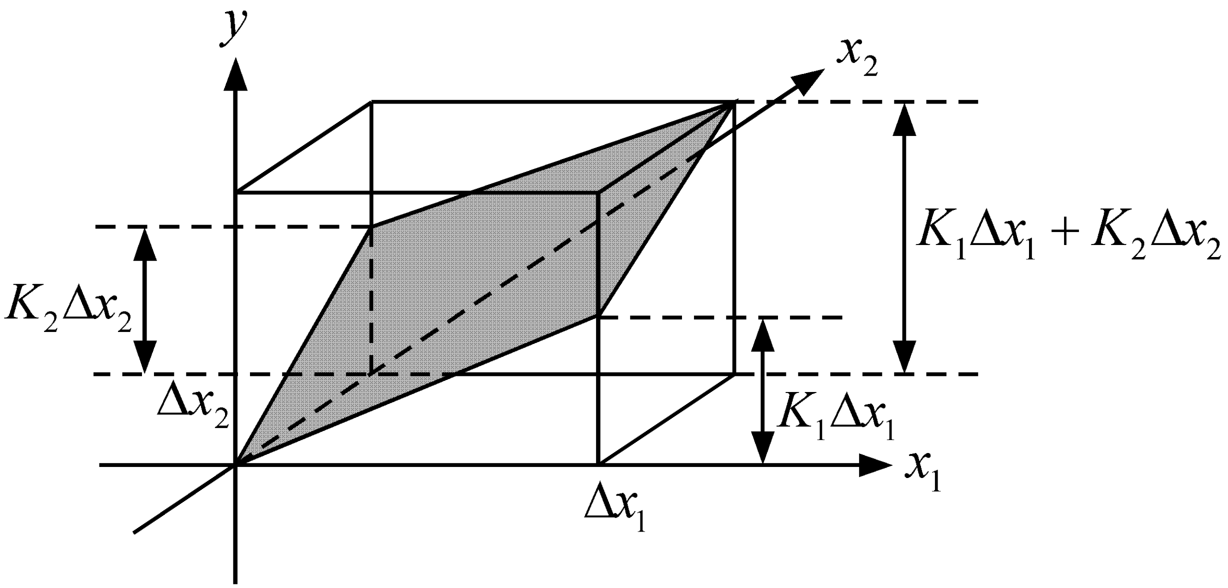
$$y = f\left(x_0 + \Delta x\right) \approx f\left(x_0\right) + \left(\left.\frac{df}{dx}\right|_{x=x_0}\right) \cdot \Delta x = f\left(x_0\right) + K \cdot \Delta x = y_0 + \Delta y$$



$$\Delta y = \left(\left. \frac{df}{dx} \right|_{x=x_0} \right) \cdot \Delta x = K \cdot \Delta x$$

2차원 선형 함수

$$y = K_1 x_1 + K_2 x_2 = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



2차원 비선형 함수

$$y = f\left(x_1, x_2\right)$$

- 선형화 중심점

$$y_0 = f\left(x_{10}, x_{20}\right)$$

- 근사식

$$y = f\left(x_{10} + \Delta x_1, x_{20} + \Delta x_2\right)$$

$$\approx f\left(x_{10}, x_{20}\right) + \left(\left.\frac{\partial f}{\partial x_1}\right|_{x_1 = x_{10}, x_2 = x_{20}}\right) \cdot \Delta x_1 + \left(\left.\frac{\partial f}{\partial x_2}\right|_{x_1 = x_{10}, x_2 = x_{20}}\right) \cdot \Delta x_2$$

$$= f\left(x_{10}, x_{20}\right) + K_1 \cdot \Delta x_1 + K_2 \cdot \Delta x_2 = y_0 + \Delta y$$

$$\Delta y = K_1 \cdot \Delta x_1 + K_2 \cdot \Delta x_2 = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

비선형 상태 변수 방정식의 선형화

- 비선형 상태 변수 방정식

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} f_1(x_1, x_2, \cdots, x_n, u) \\ f_2(x_1, x_2, \cdots, x_n, u) \\ \vdots \\ f_n(x_1, x_2, \cdots, x_n, u) \end{bmatrix}$$

- 선형화 중심점: $(x_{10}, x_{20}, \cdots, u_0)$

$$\dot{x}_{i0} = f_i(x_{10}, x_{20}, \cdots, u_0), \ i = 1, 2, \cdots, n$$

$$x_i = x_{i0} + \Delta x_i, \ i = 1, 2, \cdots n$$

$$u = u_0 + \Delta u$$

- 근사식

$$\dot{x}_i = \dot{x}_{i0} + \Delta\dot{x}_i = f_i\left(x_1, x_2, \cdots, x_n, u\right)$$

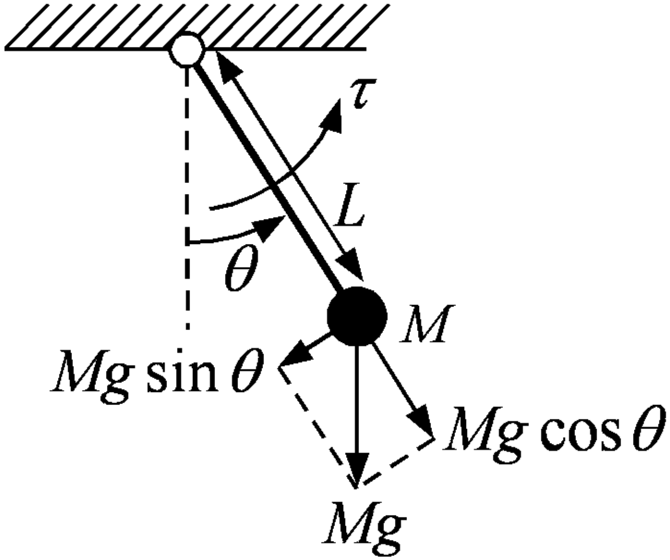
$$\begin{aligned}
&\approx f_i\left(x_{10}, x_{20}, \cdots, u_0\right) + \left(\left.\frac{\partial f_i}{\partial x_1}\right|_{x_1 = x_{10}, x_2 = x_{20}, \cdots, x_n = x_{n0}, u = u_0} \right) \cdot \Delta x_1 \\
&\quad + \left(\left.\frac{\partial f_i}{\partial x_2}\right|_{x_1 = x_{10}, x_2 = x_{20}, \cdots, x_n = x_{n0}, u = u_0} \right) \cdot \Delta x_2 \\
&\quad \vdots \\
&\quad + \left(\left.\frac{\partial f_i}{\partial x_n}\right|_{x_1 = x_{10}, x_2 = x_{20}, \cdots, x_n = x_{n0}, u = u_0} \right) \cdot \Delta x_n \\
&\quad + \left(\left.\frac{\partial f_i}{\partial u}\right|_{x_1 = x_{10}, x_2 = x_{20}, \cdots, x_n = x_{n0}, u = u_0} \right) \cdot \Delta u, \;\; i = 1, 2, \cdots, n
\end{aligned}$$

▪ 선형 상태 변수 방정식

$$\begin{bmatrix} \Delta\dot{x}_1 \\ \Delta\dot{x}_2 \\ \vdots \\ \Delta\dot{x}_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \Delta u$$

$$a_{ij} = \left.\frac{\partial f_i}{\partial x_j}\right|_{x_1 = x_{10}, x_2 = x_{20}, \cdots, x_n = x_{n0}, u = u_0}, b_i = \left.\frac{\partial f_i}{\partial u}\right|_{x_1 = x_{10}, x_2 = x_{20}, \cdots, x_n = x_{n0}, u = u_0}$$

예제



$$\tau - LMg\sin\theta = ML^2 \frac{d^2\theta}{dt^2}$$

$$x_1 = \theta,\ x_2 = \dot{\theta},\ u = \tau$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} f_1(x_1, x_2, u) \\ f_2(x_1, x_2, u) \end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{g}{L}\sin x_1 + \frac{1}{ML^2}u \end{bmatrix}$$

예제 3-27

$$\left.\frac{\partial f_1}{\partial x_1}\right|_{x_1=0,x_2=0,u=0} = 0,$$

$$\left.\frac{\partial f_1}{\partial x_2}\right|_{x_1=0,x_2=0,u=0} = 1,$$

$$\left.\frac{\partial f_2}{\partial x_1}\right|_{x_1=0,x_2=0,u=0} = -\frac{g}{L}\cos x_1\Big|_{x_1=0,x_2=0,u=0} = -\frac{g}{L},$$

$$\left.\frac{\partial f_2}{\partial x_2}\right|_{x_1=0,x_2=0,u=0} = 0$$

$$\left.\frac{\partial f_1}{\partial u}\right|_{x_1=0,x_2=0,u=0} = 0,$$

$$\left.\frac{\partial f_2}{\partial u}\right|_{x_1=0,x_2=0,u=0} = \frac{1}{ML^2}$$

예제

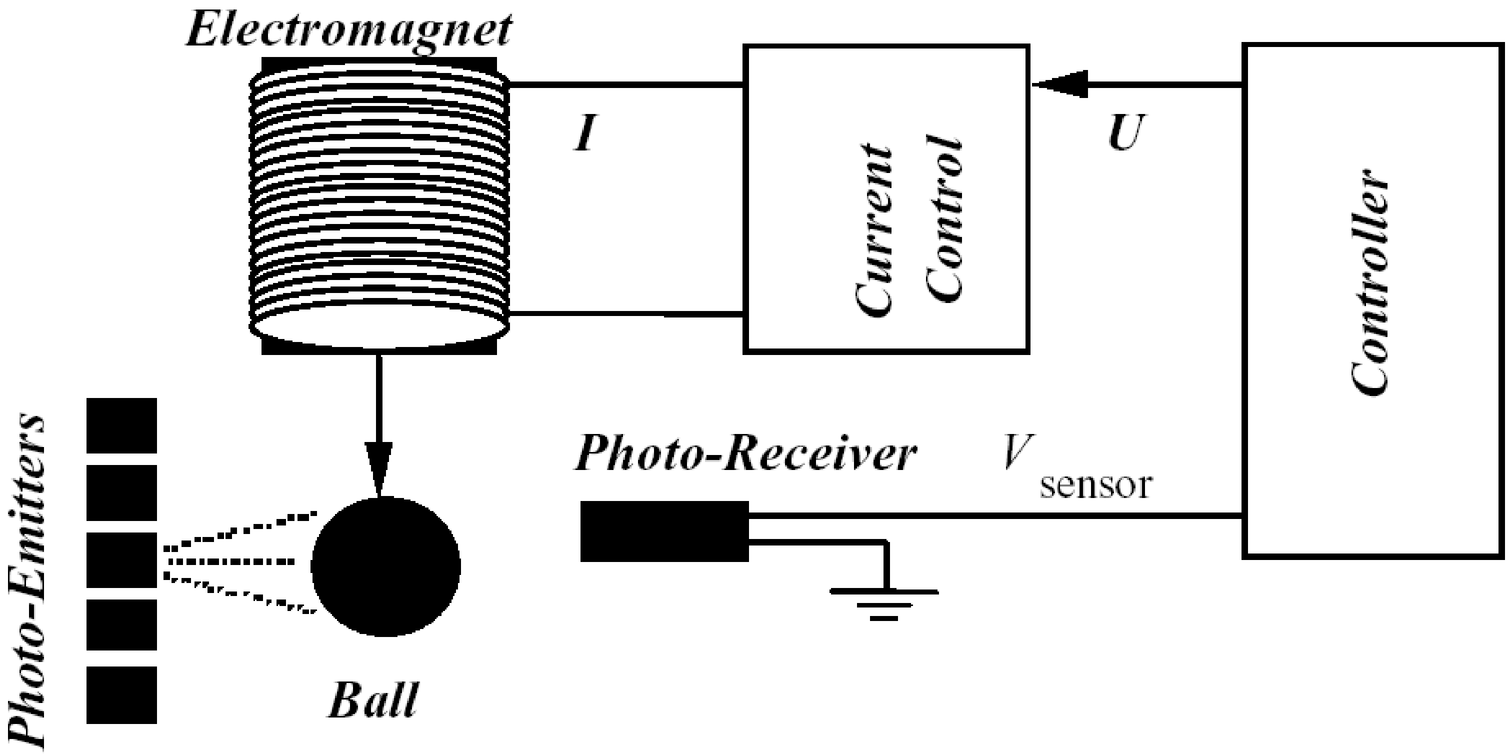
$$\begin{bmatrix} \Delta\dot{x}_1 \\ \Delta\dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{g}{L} & 0 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{ML^2} \end{bmatrix} \Delta u$$

$$s\Delta X_1(s) = \Delta X_2(s)$$

$$s\Delta X_2(s) = -\frac{g}{L}\Delta X_1(s) + \frac{1}{ML^2}\Delta U(s)$$

$$\frac{\Delta X_1(s)}{\Delta U(s)} = \frac{1}{ML^2}\frac{1}{s^2 + (g/L)}$$

Magnetic Levitation System



Dynamic Equation

$$m\ddot{x} = mg - k\frac{i^2}{x^2}$$

$$i = 0.15u + I_0$$

$$v = \gamma(x - X_0)$$

m: ball mass

g: gravitational acceleration

γ: sensor gain

x: ball position from the magnet

i: input current for magnet

v: sensor output

u: voltage input for the current amplifier

$$k = \frac{mgX_0^2}{I_0^2}$$

State Equation

$$x_1 = v$$

$$x_2 = \dot{v} = \gamma \dot{x}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} f_1(x_1, x_2, u) \\ f_2(x_1, x_2, u) \end{bmatrix} = \begin{bmatrix} x_2 \\ \gamma g - \dfrac{\gamma k}{m} \dfrac{(0.15u + I_0)^2}{(x_1/\gamma + X_0)^2} \end{bmatrix}$$

Lyapunov Linearization

$$x_1 = 0, x_2 = 0, u = 0$$

$$\begin{bmatrix} \Delta\dot{x}_1 \\ \Delta\dot{x}_2 \end{bmatrix} = A \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} + B\Delta u$$

$$A = \left. \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} \right|_{x_1 = 0, x_2 = 0, u = 0}$$

$$= \left. \begin{bmatrix} 0 & 1 \\ \frac{2k}{m}(0.15u + I_0^2)^2 (\frac{x_1}{\gamma} + X_0)^{-3} & 0 \end{bmatrix} \right|_{x_1 = 0, x_2 = 0, u = 0} = \begin{bmatrix} 0 & 1 \\ \frac{2g}{X_0} & 0 \end{bmatrix}$$

Linearization

$$B = \begin{bmatrix} \dfrac{\partial f_1}{\partial u} \\ \dfrac{\partial f_2}{\partial u} \end{bmatrix} \Bigg|_{x_1 = 0, x_2 = 0, u = 0}$$

$$= \begin{bmatrix} 0 \\ -\dfrac{0.3\gamma k}{m} \dfrac{(0.15u + I_0)}{(x_1/\gamma + X_0)^2} \end{bmatrix} \Bigg|_{x_1 = 0, x_2 = 0, u = 0} = \begin{bmatrix} 0 \\ -\dfrac{0.3\gamma g}{I_0} \end{bmatrix}$$

$$G(s) = -\frac{\eta}{s^2 - \omega_0^2}$$

$$= -\frac{1200}{s^2 - 852}$$

$$\eta = \frac{0.3\gamma g}{I_0}$$

$$\omega_0 = \sqrt{\frac{2g}{X_0}}$$

Parameters

$$\gamma = 1000/3 \;\; V/m$$

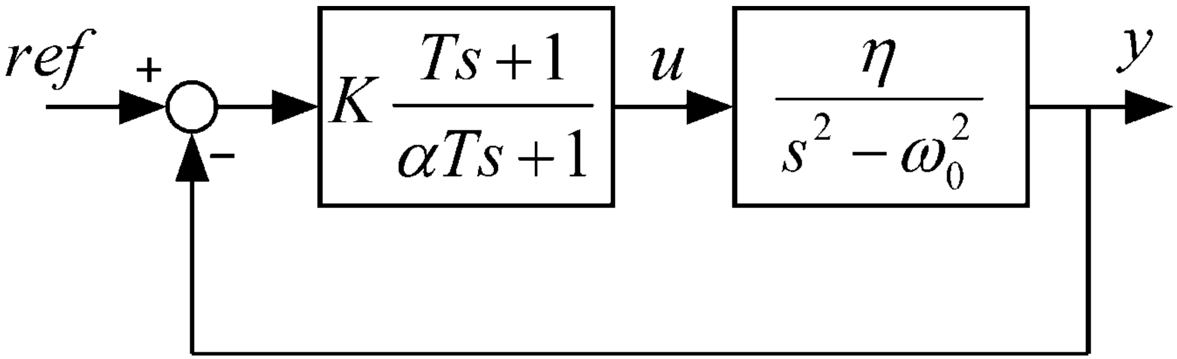
$$g = 9.8 \;\; m/\sec^2$$

$$I_0 = 0.817 \;\; A$$

$$X_0 = 23/1000 \;\; m$$

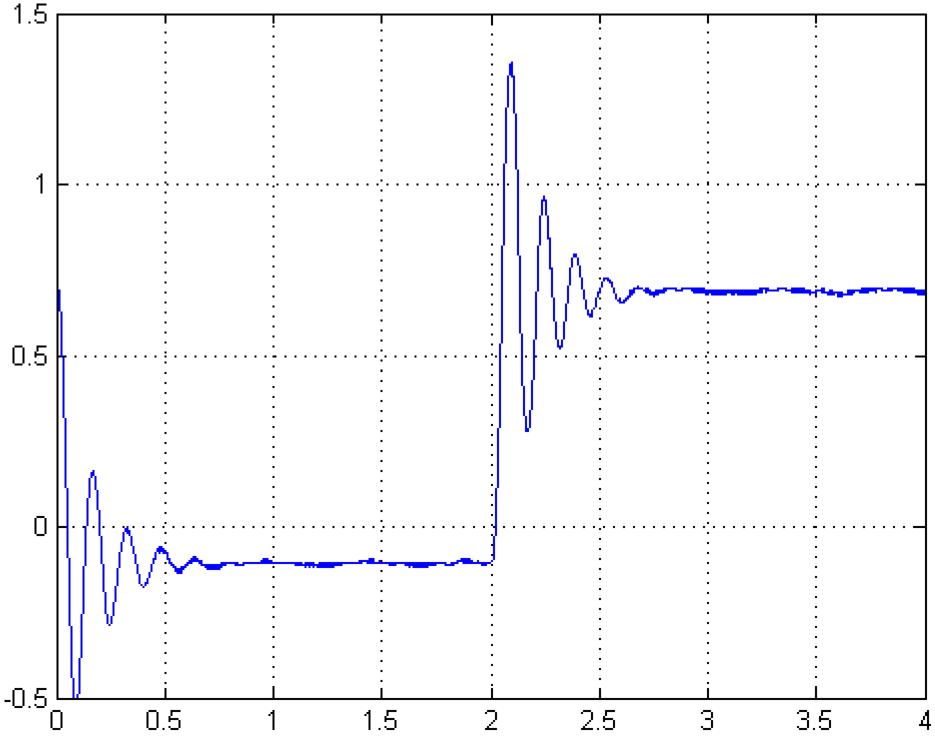
$$m = 21/1000 \; Kg$$

Lead Compensator



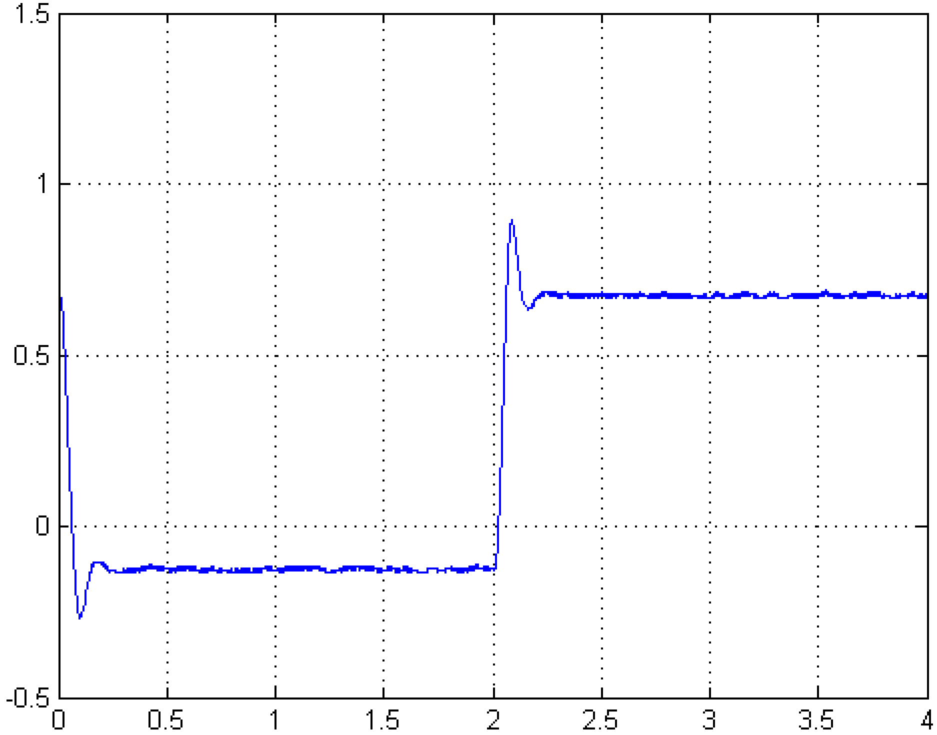
Lead Compensator

$$D_1(s) = 2\frac{0.01s+1}{0.001s+1}$$



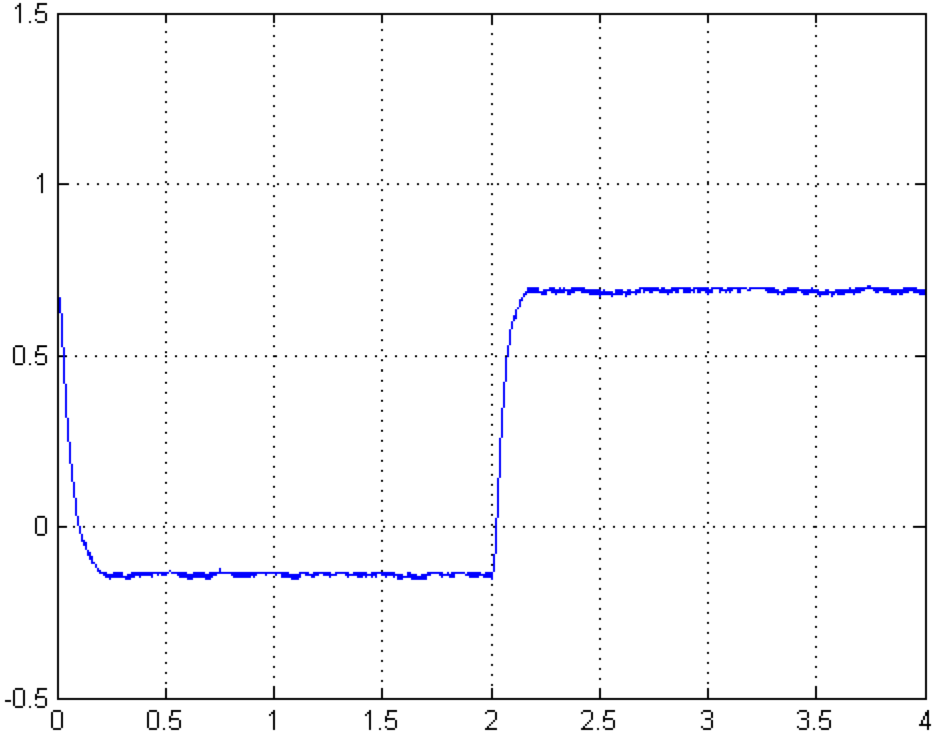
Lead Compensator

$$D_2(s) = 2\frac{0.02s+1}{0.001s+1}$$



Lead Compensator

$$D_3(s) = 2\frac{0.04s+1}{0.001s+1}$$



Feedback Linearization

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} f_1(x_1, x_2, u) \\ f_2(x_1, x_2, u) \end{bmatrix} = \begin{bmatrix} x_2 \\ \gamma g - \dfrac{\gamma k}{m} \dfrac{(0.15u + I_0)^2}{(x_1/\gamma + X_0)^2} \end{bmatrix}$$

$$w = \gamma g - \frac{\gamma k}{m} \frac{(0.15u + I_0)^2}{(x_1/\gamma + X_0)^2}$$

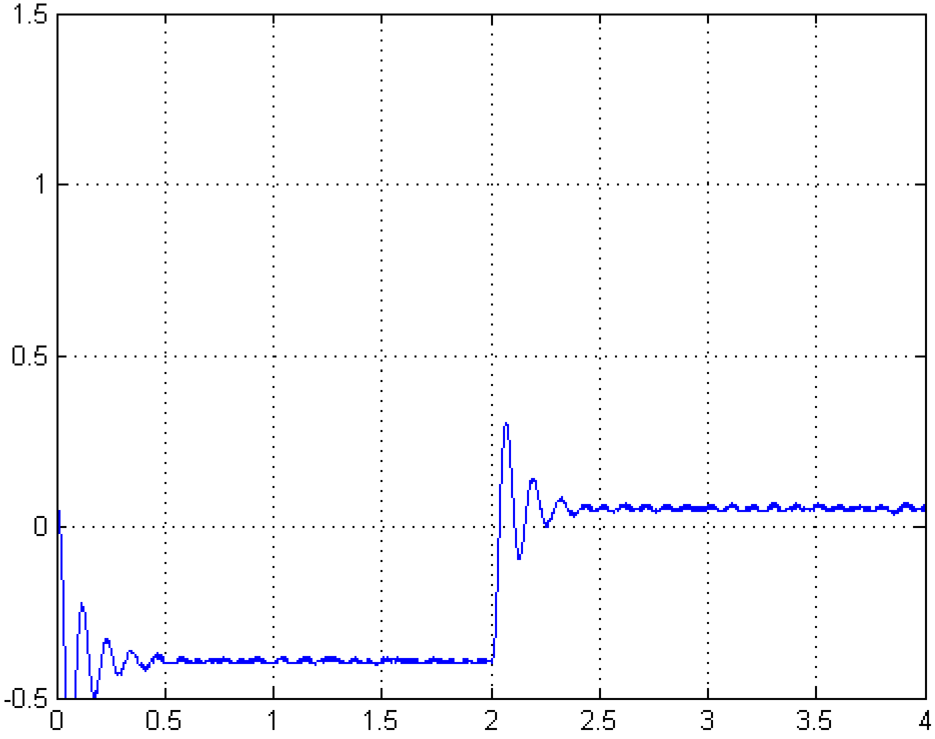
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ w \end{bmatrix}$$

$$w = k_1(ref - x_1) - k_2 x_2$$

$$u = \left(\sqrt{(\gamma g - w) m (x_1/\gamma + X_0)^2 / (\gamma k)} - I_0 \right) / 0.15$$

Feedback Linearization

$$Poles: -20 \pm j50$$



Feedback Linearization

$$Poles: -50, -50$$