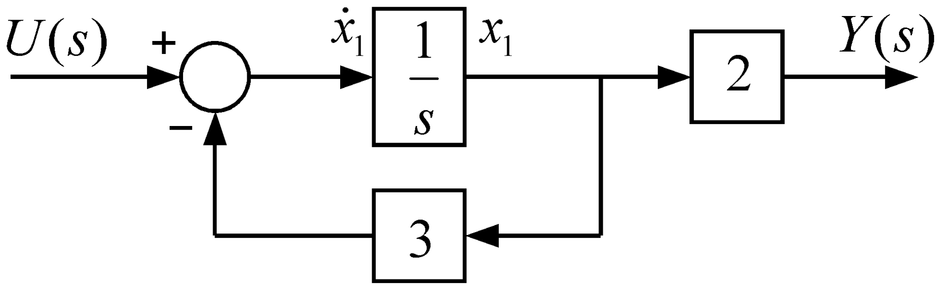
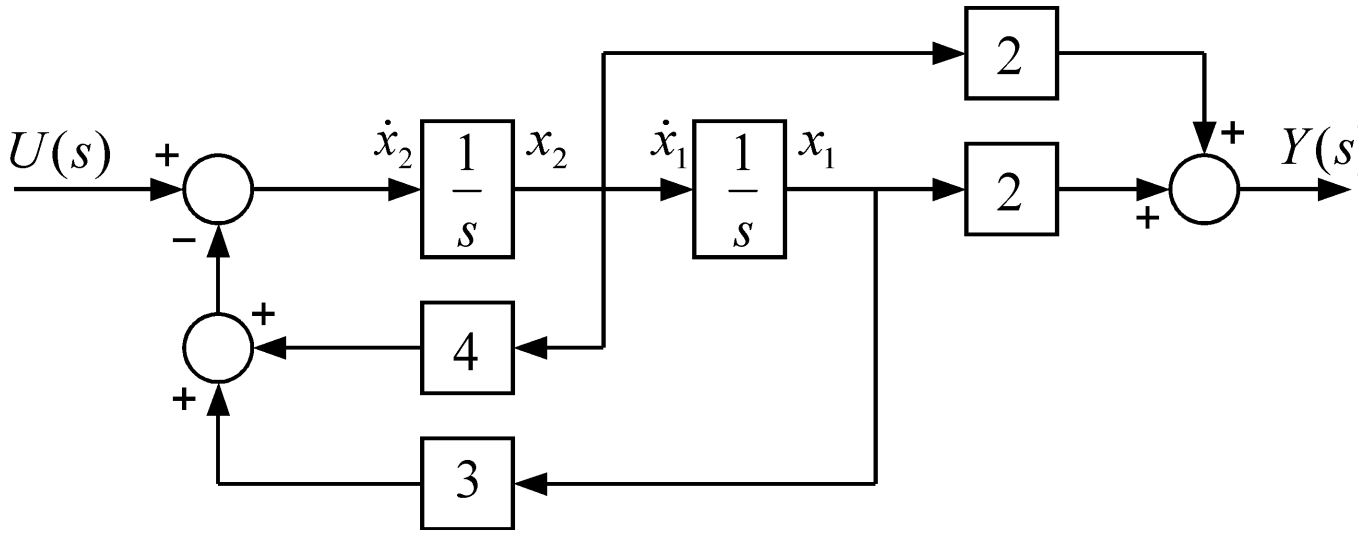
3.5 상태 변수 방정식

- 1차 시스템



$$G_1(s) = \frac{2}{s+3}$$

▶ 2차 시스템



$$G_2(s) = \frac{2s+2}{s^2+4s+3} = \frac{2(s+1)}{(s+3)(s+1)} = \frac{2}{s+3}$$

상태 변수 방정식의 예

- 미분 방정식: $\ddot{y}(t)+3\dot{y}(t)+2y(t)=u(t)$

- 상태 변수 정의: $x_1(t)=y(t)$
  $x_2(t)=\dot{y}(t)$

$$\dot{x}_1(t)=\dot{y}(t)=x_2(t)$$
$$\dot{x}_2(t)=\ddot{x}_1(t)=\ddot{y}(t)=-2y(t)-3\dot{y}(t)+u(t)=-2x_1(t)-3x_2(t)+u(t)$$

$$\dot{x}_1(t)=x_2(t)$$
$$\dot{x}_2(t)=-2x_1(t)-3x_2(t)+u(t)$$

- 상태 변수 벡터: $x(t)=\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$

- 벡터 방정식:

$$\dot{x}(t)=\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix}=\begin{bmatrix} x_2(t) \\ -2x_1(t)-3x_2(t)+u(t) \end{bmatrix}=\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}+\begin{bmatrix} 0 \\ 1 \end{bmatrix}u(t)$$

$$A=\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}, B=\begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \dot{x}(t)=Ax(t)+Bu(t)$$

$$y(t)=x_1(t)=\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}=\begin{bmatrix} 1 & 0 \end{bmatrix}x(t)$$

예제 3-18

- 1차 시스템

$$\dot{x}_1 = -3x_1 + u$$
$$y = 2x_1$$

- 2차 시스템

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -3x_1 - 4x_2 + u$$
$$y = 2x_1 + 2x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

상태 벡터, 상태 변수 방정식

$$x(t) \triangleq \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}$$

$$\dot{x}(t) = f(x(t), u(t))$$

$$y(t) = g(x(t), u(t))$$

선형 시불변 시스템

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

- 단일-입력 단일-출력 시스템
- 시스템 행렬, 입력 행렬

$$x:(n\times 1), u:(1\times 1), y:(1\times 1)$$

$$A:(n\times n), B:(n\times 1), C:(1\times n), D:(1\times 1)$$

제어 가능 표준형
(Controllable Canonical Form)

$$\frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}} + \cdots + a_1\frac{dy(t)}{dt} + a_0 y(t) = u(t)$$

$$x_1(t) = y(t)$$

$$x_2(t) = y^{(1)}(t)$$

$$\vdots$$

$$x_n(t) = y^{(n-1)}(t)$$

제어 가능 표준형 (Controllable Canonical Form)

$$\dot{x}_1(t) = y^{(1)}(t) = x_2(t)$$
$$\dot{x}_2(t) = y^{(2)}(t) = x_3(t)$$
$$\vdots$$
$$\dot{x}_n(t) = y^{(n)}(t) = -a_0 x_1(t) - a_1 x_2(t) - \cdots - a_{n-1} x_n(t) + u(t)$$

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix}, D = 0$$

제어 가능 표준형
(Controllable Canonical Form)

$$\frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}} + \cdots + a_1\frac{dy(t)}{dt} + a_0 y(t)$$

$$= b_{n-1}\frac{d^{n-1}u(t)}{dt^{n-1}} + b_{n-2}\frac{d^{n-2}u(t)}{dt^{n-2}} + \cdots + b_1\frac{du(t)}{dt} + b_0 u(t)$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \cdot\cdot\cdot + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdot\cdot\cdot + a_1 s + a_0}$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \cdot\cdot\cdot + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdot\cdot\cdot + a_1 s + a_0} \cdot \frac{Z(s)}{Z(s)}$$

제어 가능 표준형
(Controllable Canonical Form)

$$U(s) = \left(s^n + a_{n-1}s^{n-1} + \cdot \cdot \cdot + a_1 s + a_0\right)Z(s)$$

$$Y(s) = \left(b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \cdot \cdot \cdot + b_1 s + b_0\right)Z(s)$$

$$\frac{d^n z(t)}{dt^n} + a_{n-1}\frac{d^{n-1}z(t)}{dt^{n-1}} + \cdot \cdot \cdot + a_1\frac{dz(t)}{dt} + a_0 z(t) = u(t)$$

$$x_1(t) = z(t)$$

$$x_2(t) = z^{(1)}(t)$$

$$\vdots$$

$$x_n(t) = z^{(n-1)}(t)$$

제어 가능 표준형 (Controllable Canonical Form)

$$\dot{x}_1(t) = z^{(1)}(t) = x_2(t)$$

$$\dot{x}_2(t) = z^{(2)}(t) = x_3(t)$$

$$\vdots$$

$$\dot{x}_n(t) = z^{(n)}(t) = -a_0 x_1(t) - a_1 x_2(t) - \cdots - a_{n-1} x_n(t) + u(t)$$

$$y(t) = b_{n-1}\frac{d^{n-1}z(t)}{dt^{n-1}} + b_{n-2}\frac{d^{n-2}z(t)}{dt^{n-2}} + \cdots + b_1\frac{dz(t)}{dt} + b_0 z(t)$$

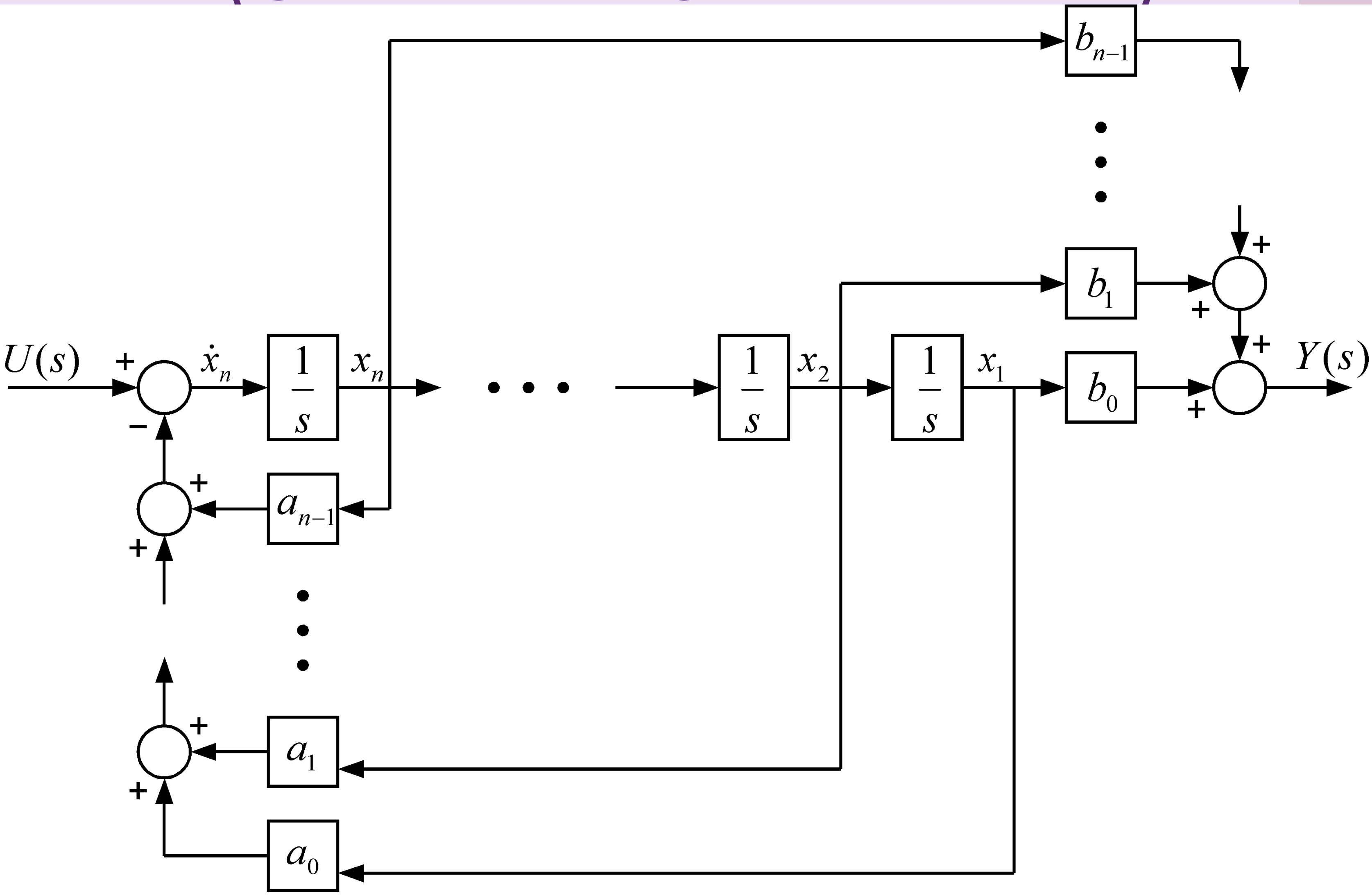
$$= b_{n-1}x_n(t) + b_{n-2}x_{n-1}(t) + \cdots + b_1 x_2(t) + b_0 x_1(t)$$

제어 가능 표준형
(Controllable Canonical Form)

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} u
$$

$$
y = \begin{bmatrix} b_0 & b_1 & \cdots & b_{n-2} & b_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}
$$

제어 가능 표준형
(Controllable Canonical Form)



예제 3-19

$$G(s)=\frac{Y(s)}{U(s)}=\frac{1}{s^3+2s^2+3s+1}$$

$$\left(s^3+2s^2+3s+1\right)Y(s)=U(s)$$

$$\frac{d^3y(t)}{dt^3}+2\frac{d^2y(t)}{dt^2}+3\frac{dy(t)}{dt}+y(t)=u(t)$$

$$x_1(t)=y(t)$$

$$x_2(t)=y^{(1)}(t)=\frac{dy(t)}{dt}$$

$$x_3(t)=y^{(2)}(t)=\frac{d^2y(t)}{dt^2}$$

예제 3-19

$$\dot{x}_1(t) = y^{(1)}(t) = x_2(t)$$

$$\dot{x}_2(t) = y^{(2)}(t) = x_3(t)$$

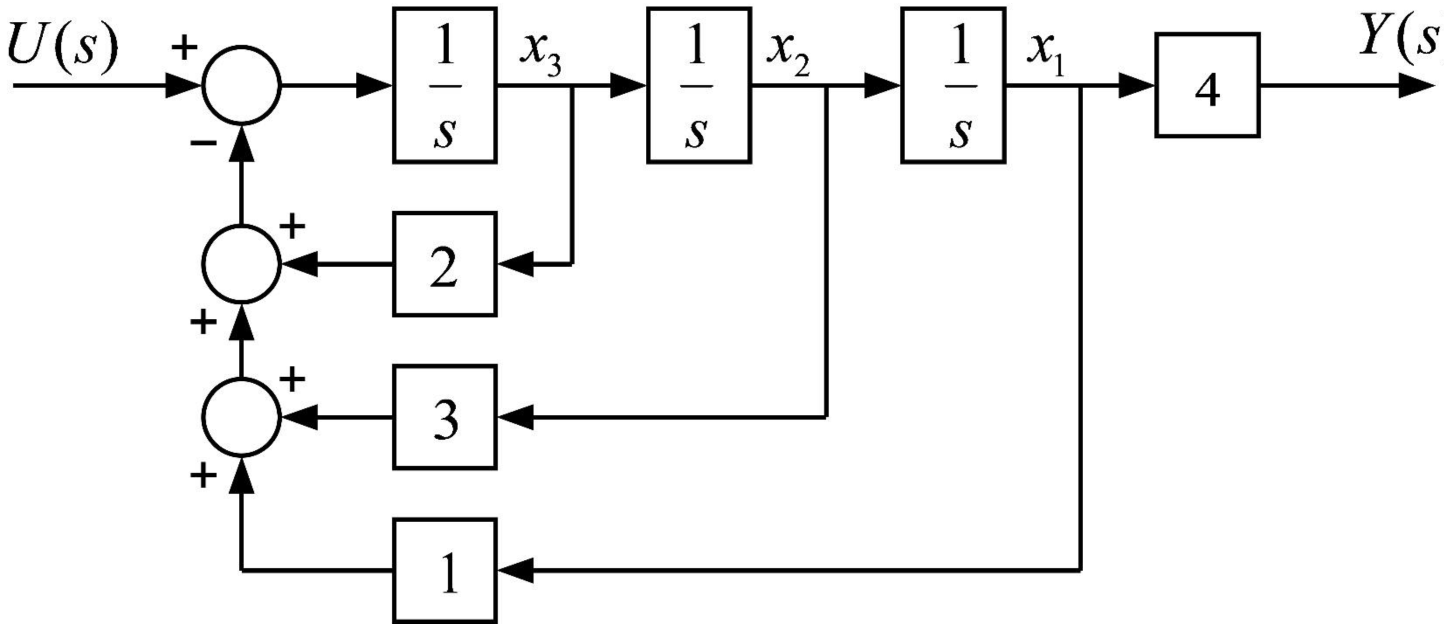
$$\dot{x}_3(t) = y^{(3)}(t) = \frac{d^3y(t)}{dt^3} = -2\frac{d^2y(t)}{dt^2} - 3\frac{dy(t)}{dt} - y(t) + u(t)$$

$$= -x_1(t) - 3x_2(t) - 2x_3(t) + u(t)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

예제 3-19



예제 3-20

$$G(s)=\frac{Y(s)}{U(s)}=\frac{5s+4}{s^3+2s^2+3s+1}$$

$$\frac{Y(s)}{U(s)}=\frac{5s+4}{s^3+2s^2+3s+1}\cdot\frac{Z(s)}{Z(s)}$$

$$U(s)=\left(s^3+2s^2+3s+1\right)Z(s)$$

$$Y(s)=\left(5s+4\right)Z(s)$$

$$x_1=z, x_2=\dot{z}, x_3=\ddot{z}$$

예제 3-20

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

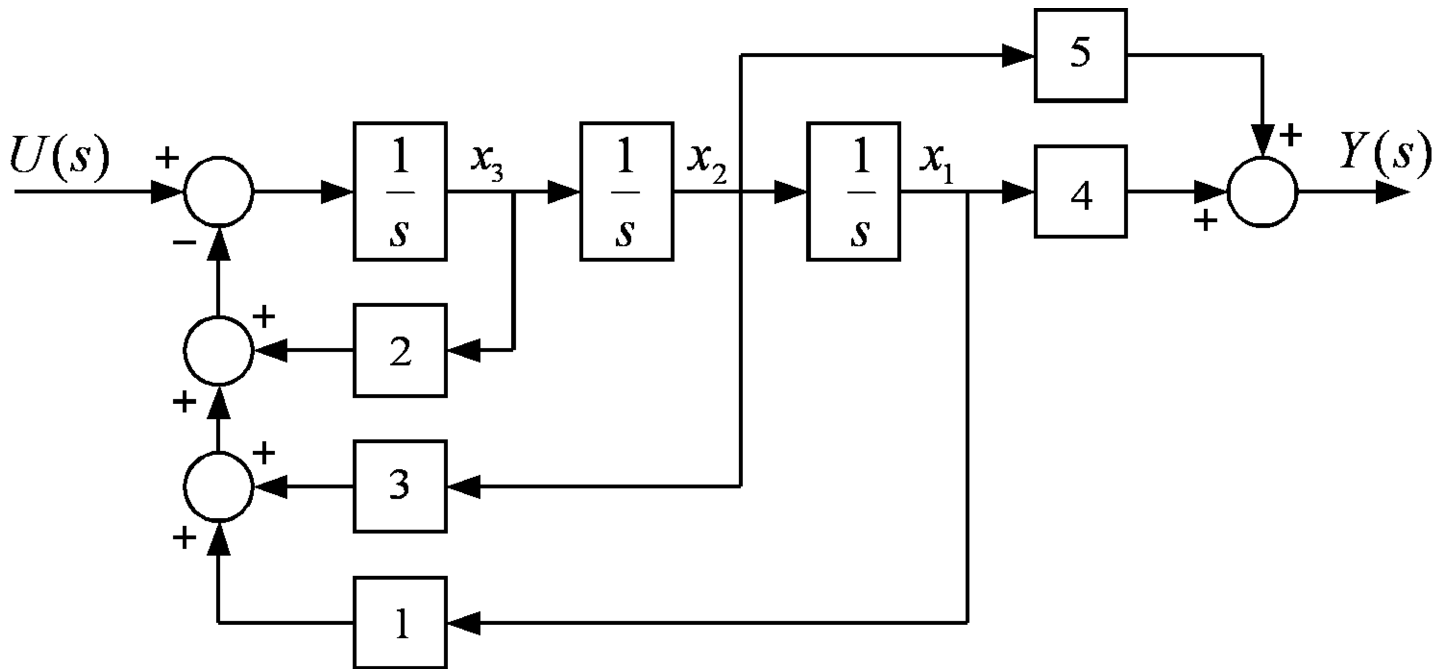
$$\dot{x}_3 = \dddot{z} = -x_1 - 3x_2 - 2x_3 + u$$

$$y = 4x_1 + 5x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 4 & 5 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

예제 3-20



관측 가능 표준형
(Observable Canonical Form)

$$\frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}} + \cdots + a_1\frac{dy(t)}{dt} + a_0 y(t)$$

$$= b_{n-1}\frac{d^{n-1}u(t)}{dt^{n-1}} + \cdots + b_1\frac{du(t)}{dt} + b_0 u(t)$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \cdot\cdot\cdot + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdot\cdot\cdot + a_1 s + a_0}$$

$$\left(s^n + a_{n-1}s^{n-1} + \cdot\cdot\cdot + a_1 s + a_0\right)Y(s)$$

$$= \left(b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \cdot\cdot\cdot + b_1 s + b_0\right)U(s)$$

관측 가능 표준형 (Observable Canonical Form)

$$b_0 U(s) - a_0 Y(s) = \left(s^n + a_{n-1}s^{n-1} + \cdot\ \cdot\ \cdot + a_1 s\right)Y(s)$$

$$-\left(b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \cdot\ \cdot\ \cdot + b_1 s\right)U(s) = sX_1(s)$$

$$X_1(s) = \left(s^{n-1} + a_{n-1}s^{n-2} + \cdot\ \cdot\ \cdot + a_1\right)Y(s)$$

$$-\left(b_{n-1}s^{n-2} + b_{n-2}s^{n-3} + \cdot\ \cdot\ \cdot + b_1\right)U(s)$$

$$X_1(s) + b_1 U(s) - a_1 Y(s) = \left(s^{n-1} + a_{n-1}s^{n-2} + \cdot\ \cdot\ \cdot + a_2 s\right)Y(s)$$

$$-\left(b_{n-1}s^{n-2} + b_{n-2}s^{n-3} + \cdot\ \cdot\ \cdot + b_2 s\right)U(s) = sX_2(s)$$

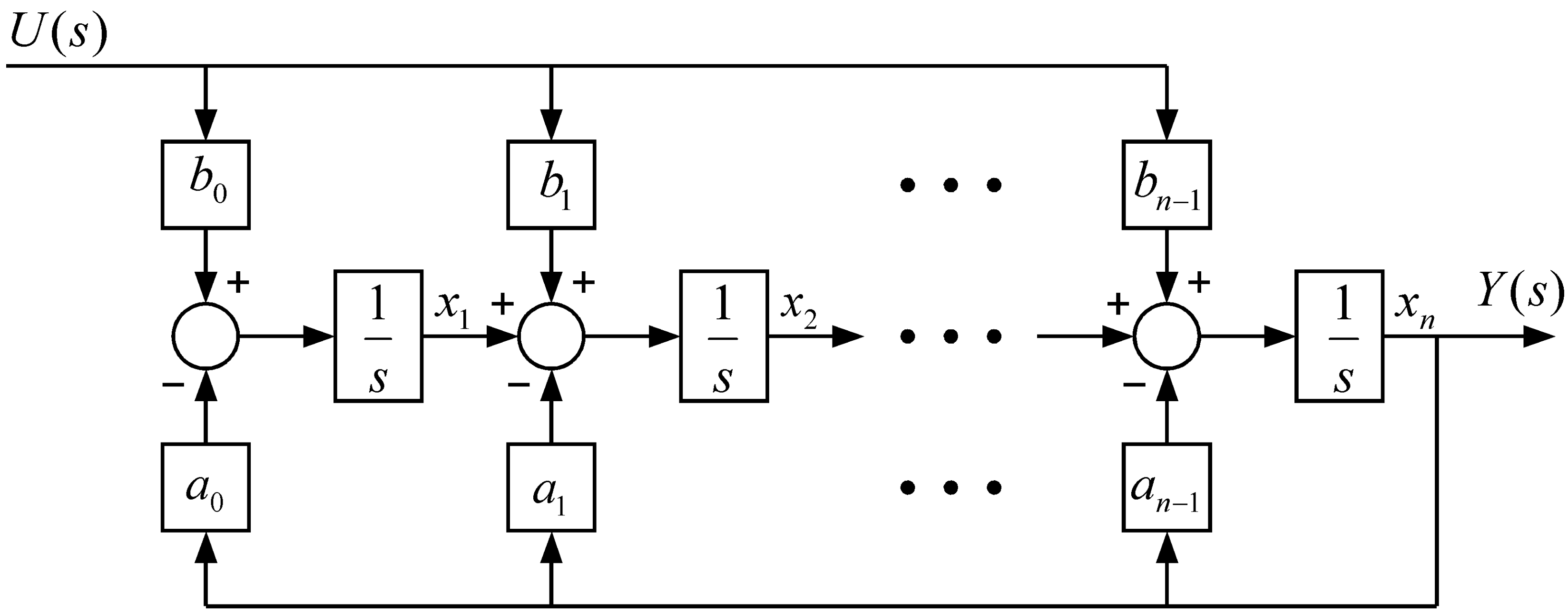
관측 가능 표준형 (Observable Canonical Form)

$$X_2(s) = \left(s^{n-2} + a_{n-1}s^{n-3} + \cdot\ \cdot\ \cdot + a_2\right)Y(s)$$

$$-\left(b_{n-1}s^{n-3} + b_{n-2}s^{n-4} + \cdot\ \cdot\ \cdot + b_2\right)U(s)$$

$$X_{n-1}(s) + b_{n-1}U(s) - a_{n-1}Y(s) = sY(s) = sX_n(s)$$

관측 가능 표준형
(Observable Canonical Form)



관측 가능 표준형
(Observable Canonical Form)

$$\dot{x}_1 = -a_0 x_n + b_0 u$$
$$\dot{x}_2 = x_1 - a_1 x_n + b_1 u$$
$$\vdots$$
$$\dot{x}_n = x_{n-1} - a_{n-1} x_n + b_{n-1} u$$
$$y = x_n$$

관측 가능 표준형 (Observable Canonical Form)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 0 & \cdots & 0 & -a_0 \\ 1 & 0 & \cdots & 0 & -a_1 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_{n-1} \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

예제 3-21

$$G(s)=\frac{Y(s)}{U(s)}=\frac{5s+4}{s^3+2s^2+3s+1}$$

$$\left(s^3+2s^2+3s+1\right)Y(s)=\left(5s+4\right)U(s)$$

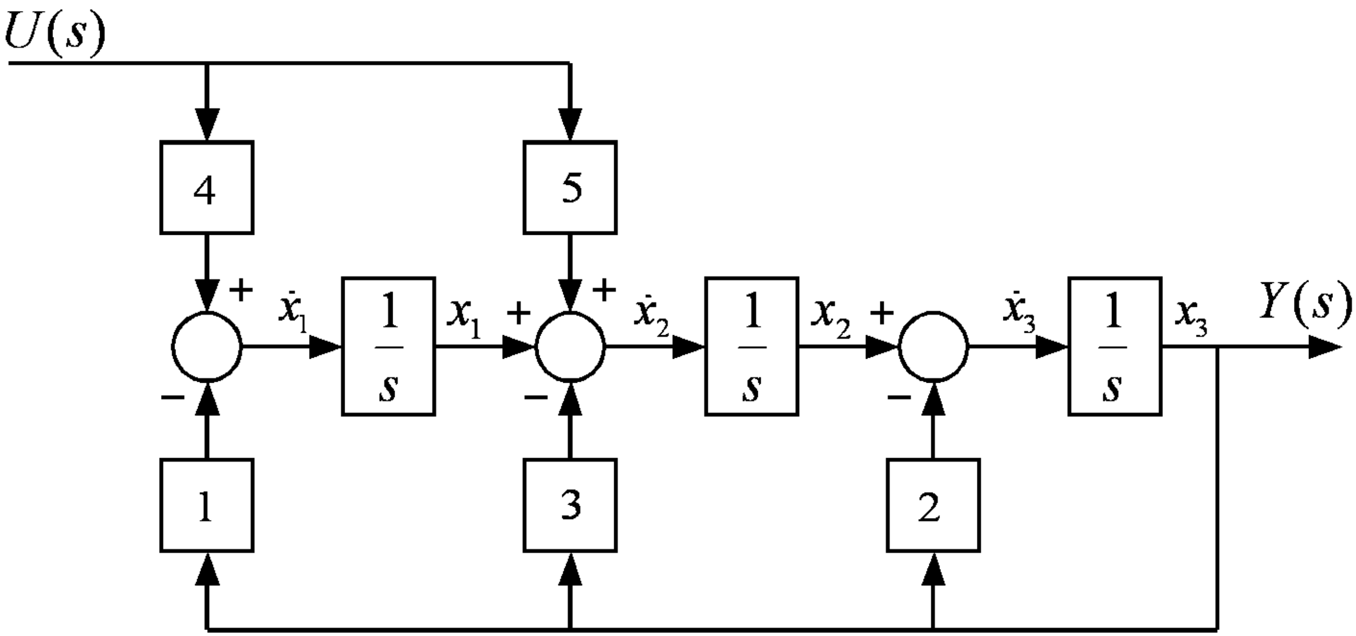
$$4U(s)-Y(s)=\left(s^3+2s^2+3s\right)Y(s)-5sU(s)=sX_1(s)$$

$$X_1(s)+5U(s)-3Y(s)=\left(s^2+2s\right)Y(s)=sX_2(s)$$

$$X_2(s)-2Y(s)=sY(s)=sX_3(s)$$

$$Y(s)=X_3(s)$$

예제 3-21



예제 3-21

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & -3 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 4 \\ 5 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

예제 3-22

$$G(s) = \frac{Y(s)}{U(s)} = \frac{2s^3 + 4s^2 + 11s + 6}{s^3 + 2s^2 + 3s + 1}$$

$$G(s) = 2 + \frac{5s + 4}{s^3 + 2s^2 + 3s + 1}$$

$$\mathcal{L}\left[y(t)\right] = Y(s) = G(s)U(s) = 2U(s) + \frac{5s + 4}{s^3 + 2s^2 + 3s + 1}U(s)$$

$$Y(s) = Y_1(s) + Y_2(s)$$

$$Y_1(s) = \mathcal{L}\left[y_1(t)\right] = 2U(s)$$

$$Y_2(s) = \mathcal{L}\left[y_2(t)\right] = \frac{5s + 4}{s^3 + 2s^2 + 3s + 1}U(s)$$

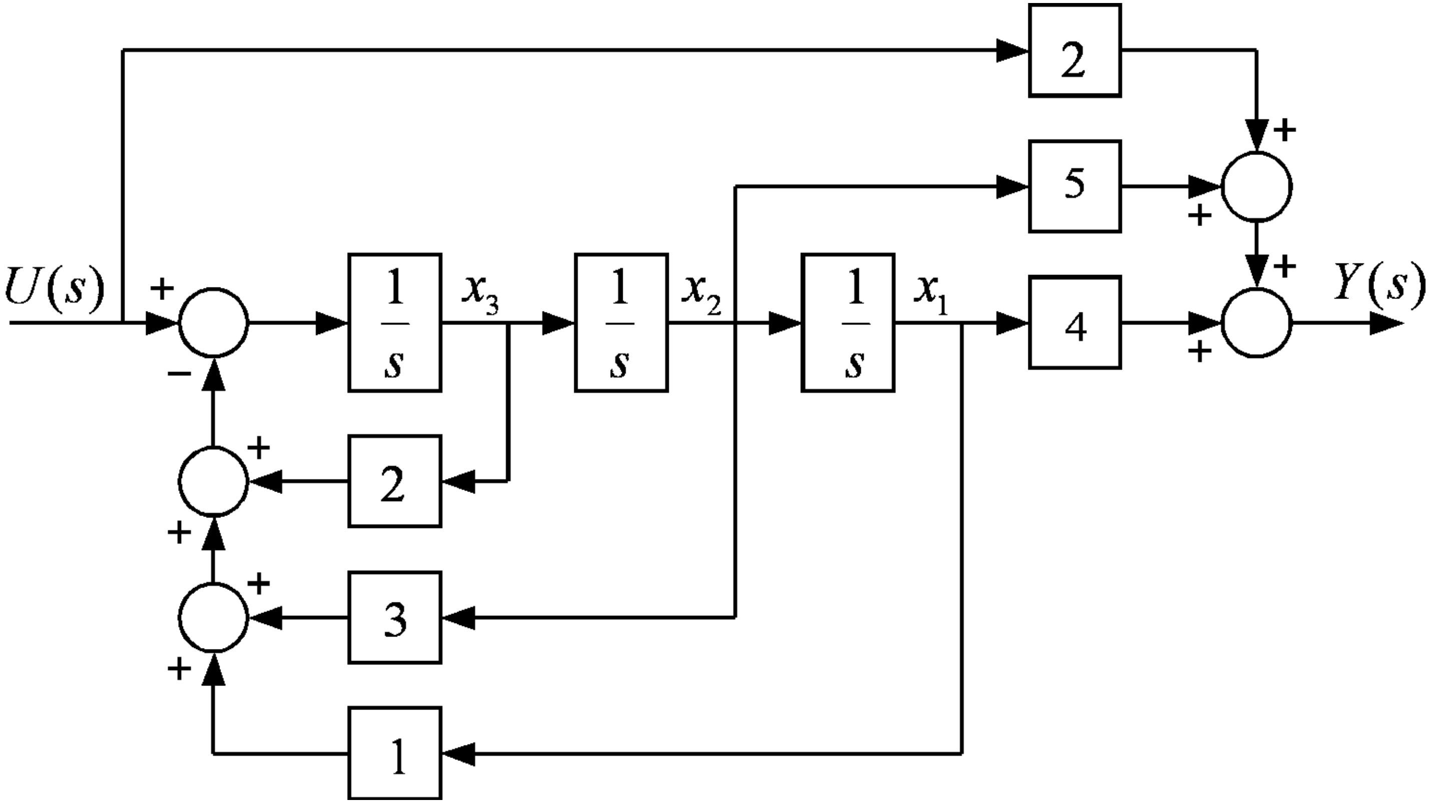
예제 3-22

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y_2 = \begin{bmatrix} 4 & 5 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$y = y_1 + y_2 = \begin{bmatrix} 4 & 5 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 2u$$

예제 3-22

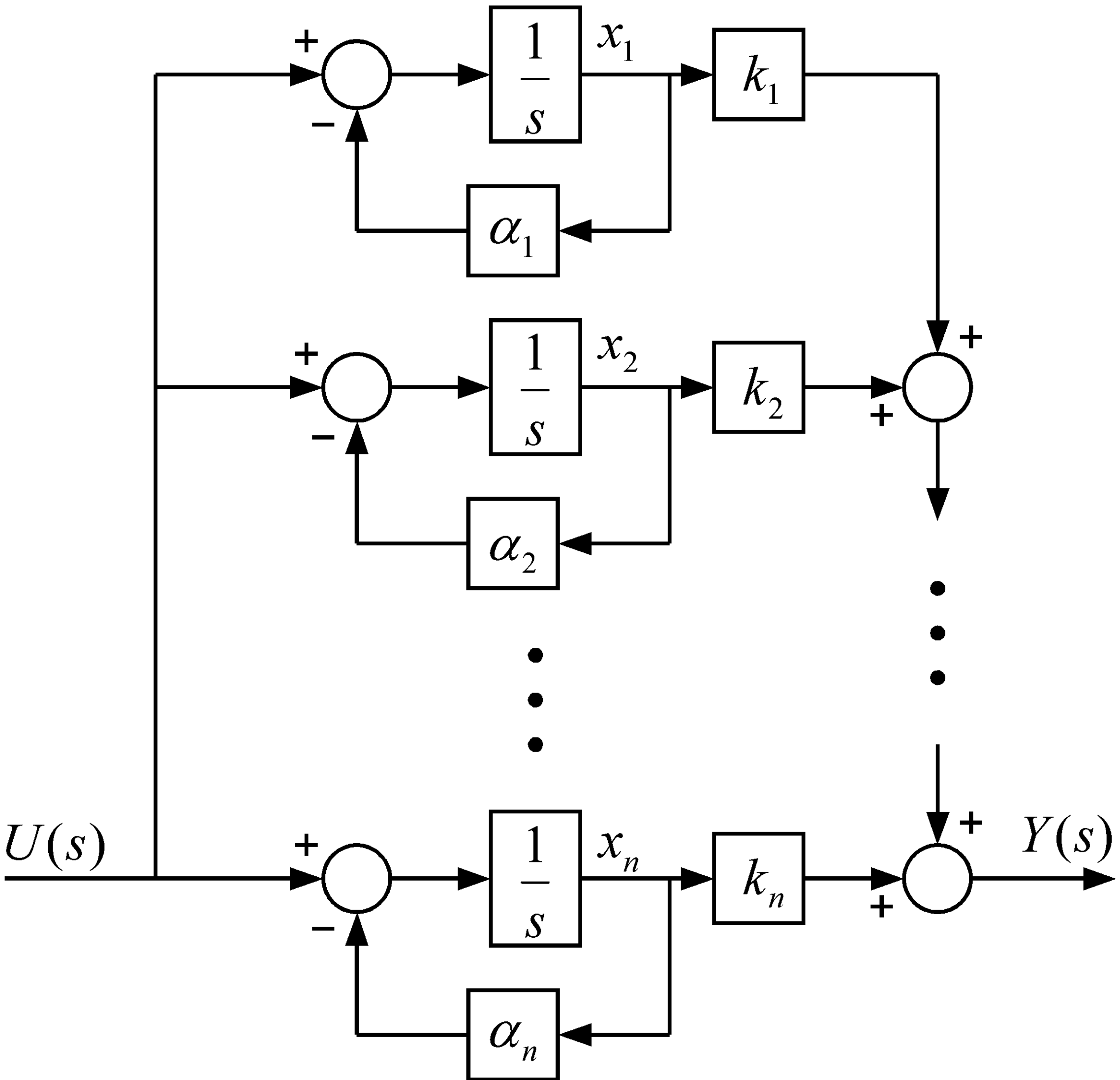


대각 표준형
(Diagonal Canonical Form)

$$G(s)=\frac{Y(s)}{U(s)}=\frac{\beta}{(s+\alpha_1)(s+\alpha_2)\cdots(s+\alpha_n)}$$

$$G(s)=\frac{k_1}{s+\alpha_1}+\frac{k_2}{s+\alpha_n}+\cdots+\frac{k_n}{s+\alpha_n}$$

대각 표준형 (Diagonal Canonical Form)



대각 표준형
(Diagonal Canonical Form)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} -\alpha_1 & 0 & \cdots & 0 \\ 0 & -\alpha_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -\alpha_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} k_1 & k_2 & \cdots & k_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

예제 3-23

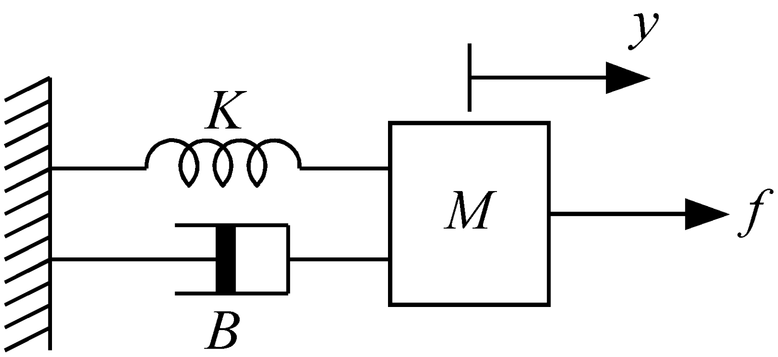
$$G(s) = \frac{1}{(s+1)(s+2)} = \frac{1}{s+1} + \frac{-1}{s+2}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

3.6 상태 변수를 이용한 동적 시스템 모델링

- 예제 3-24



$$M\frac{d^2y}{dt^2}+B\frac{dy}{dt}+Ky=f$$

$$x_1 = y$$

$$x_2 = \dot{y}$$

예제 3-24

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \ddot{y} = -\frac{K}{M}y - \frac{B}{M}\dot{y} + \frac{f}{M}$$

$$= -\frac{K}{M}x_1 - \frac{B}{M}x_2 + \frac{1}{M}u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K}{M} & -\frac{B}{M} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

예제 3-24

- 관측 가능 표준형

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} 0 & -\dfrac{K}{M} \\ 1 & -\dfrac{B}{M} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{M} \\ 0 \end{bmatrix} u$$

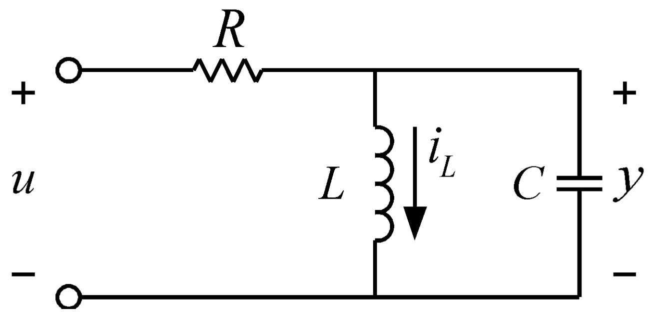
$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

$$z_2 = y = x_1$$

$$z_1 = \frac{B}{M} z_2 + \dot{z}_2 = \frac{B}{M} x_1 + \dot{x}_1 = \frac{B}{M} x_1 + x_2$$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} \dfrac{B}{M} & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

예제 3-25



$$x_1 = y$$
$$x_2 = i_L$$

$$\dot{x}_1 = \dot{y} = \frac{1}{C}\left(\frac{u-y}{R} - i_L\right) = -\frac{1}{RC}x_1 - \frac{1}{C}x_2 + \frac{1}{RC}u$$

$$\dot{x}_2 = \dot{i}_L = \frac{1}{L}y = \frac{1}{L}x_1$$

$$y = x_1$$

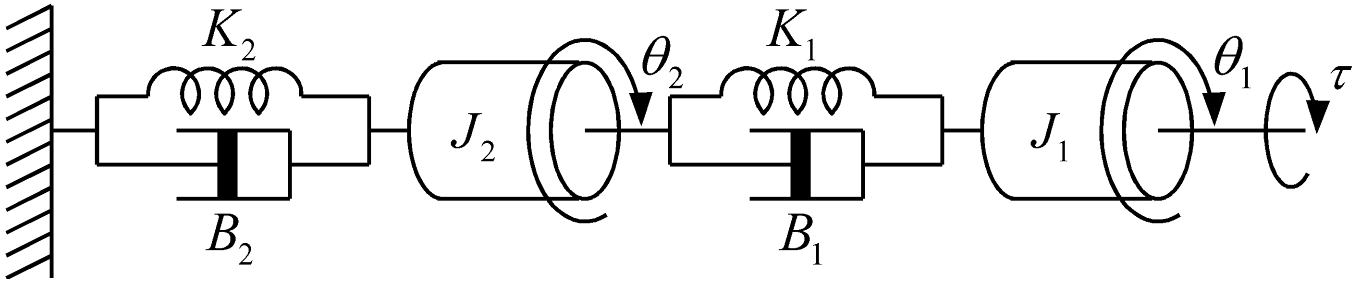
예제 3-25

$$G(s)=\frac{Y(s)}{U(s)}=\frac{\frac{s}{RC}}{s^2+\frac{s}{RC}+\frac{1}{LC}}$$

$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -\frac{1}{LC} & -\frac{1}{RC}\end{bmatrix}\begin{bmatrix}z_1\\ z_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u$$

$$y=\begin{bmatrix}0 & \frac{1}{RC}\end{bmatrix}\begin{bmatrix}z_1\\ z_2\end{bmatrix}$$

예제 3-26



$$\tau - K_1(\theta_1 - \theta_2) - B_1\left(\frac{d\theta_1}{dt} - \frac{d\theta_2}{dt}\right) = J_1\frac{d^2\theta_1}{dt^2}$$

$$K_1(\theta_1 - \theta_2) + B_1\left(\frac{d\theta_1}{dt} - \frac{d\theta_2}{dt}\right) - K_2\theta_2 - B_2\frac{d\theta_2}{dt} = J_2\frac{d^2\theta_2}{dt^2}$$

예제 3-26

$$x_1 = \theta_1, x_2 = \dot{\theta}_1, x_3 = \theta_2, x_4 = \dot{\theta}_2$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \ddot{\theta}_1 = -\frac{K_1}{J_1}x_1 - \frac{B_1}{J_1}x_2 + \frac{K_1}{J_1}x_3 + \frac{B_1}{J_1}x_4 + \frac{1}{J_1}u$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = \ddot{\theta}_2 = \frac{K_1}{J_2}x_1 + \frac{B_1}{J_2}x_2 - \left(\frac{K_1}{J_2} + \frac{K_2}{J_2}\right)x_3 - \left(\frac{B_1}{J_2} + \frac{B_2}{J_2}\right)x_4$$

$$y_1 = x_1$$

예제 3-26

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ -\dfrac{K_1}{J_1} & -\dfrac{B_1}{J_1} & \dfrac{K_1}{J_1} & \dfrac{B_1}{J_1}\\ 0 & 0 & 0 & 1\\ \dfrac{K_1}{J_2} & \dfrac{B_1}{J_2} & -\left(\dfrac{K_1}{J_2}+\dfrac{K_2}{J_2}\right) & -\left(\dfrac{B_1}{J_2}+\dfrac{B_2}{J_2}\right)\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\begin{bmatrix}0\\ \dfrac{1}{J_1}\\ 0\\ 0\end{bmatrix}u$$

$$y=\begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}$$

상태 변수 방정식과 전달 함수의 관계

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad X(s) = \begin{bmatrix} X_1(s) \\ X_2(s) \\ \vdots \\ X_n(s) \end{bmatrix}$$

$$sX(s) - x(0) = AX(s) + BU(s)$$

$$Y(s) = CX(s) + DU(s)$$

상태 변수 방정식과 전달 함수의 관계

$$X(s) = (sI - A)^{-1} x(0) + (sI - A)^{-1} BU(s)$$

$$Y(s) = C(sI - A)^{-1} x(0) + \left[C(sI - A)^{-1} B + D \right] U(s)$$

$$Y(s) = \left[C(sI - A)^{-1} B + D \right] U(s)$$

$$G(s) = C(sI - A)^{-1} B + D$$

$$Y(s) = G(s)U(s)$$

예제 8-1

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -2x_1 - 3x_2 + u$$

$$sX_1(s) = X_2(s)$$

$$sX_2(s) = -2X_1(s) - 3X_2(s) + U(s)$$

예제 8-1

$$s^2 X_1(s) = -2X_1(s) - 3sX_1(s) + U(s)$$

$$\left(s^2 + 3s + 2\right) X_1(s) = U(s)$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{X_1(s)}{U(s)} = \frac{1}{s^2 + 3s + 2}$$

예제 8-1

- 공식을 이용한 방법:

$$G(s) = C\left(sI - A\right)^{-1} B + D$$

$$\left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}\right)^{-1} = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}^{-1}$$

$$= \frac{1}{s^2 + 3s + 2}\begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}$$

$$G(s) = \frac{1}{s^2 + 3s + 2}\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{s^2 + 3s + 2}$$

예제 8-2

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x + 4u$$

$$G(s) = \frac{1}{s^2 + 3s + 2} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 4 = \frac{1}{s^2 + 3s + 2} + 4$$

$$= \frac{4s^2 + 12s + 9}{s^2 + 3s + 2}$$

상태 변수 방정식의 안정도

- 극점(pole): 전달 함수의 분모=0

$$G(s) = C\left(sI - A\right)^{-1} B + D = \frac{C \cdot \mathrm{Adj}(sI - A) \cdot B}{|sI - A|} + D$$

$$|sI - A| = \det(sI - A) = 0$$

상태 변수 방정식의 안정도

- 극점(pole)과 고윳값(eigenvalue)

$$\left| sI - A \right| = 0$$

$$Ax = sx \quad (x \neq 0)$$

$$\left(sI - A \right) x = 0$$

예제 8-3

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

$$\left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} \right| = \left| \begin{bmatrix} s & -1 \\ -2 & s+1 \end{bmatrix} \right|$$

$$= s^2 + s - 2 = (s-1)(s+2) = 0$$

$$s = 1, s = -2$$

불안정 시스템