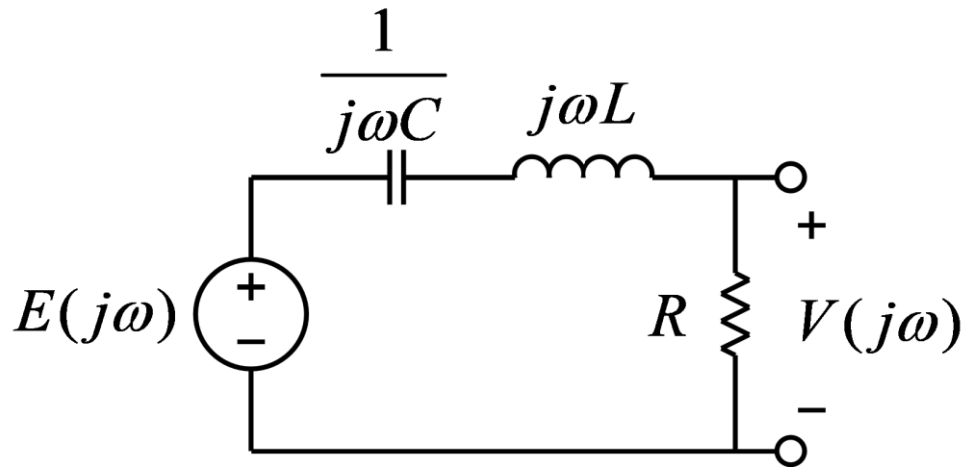

Resonant Circuits

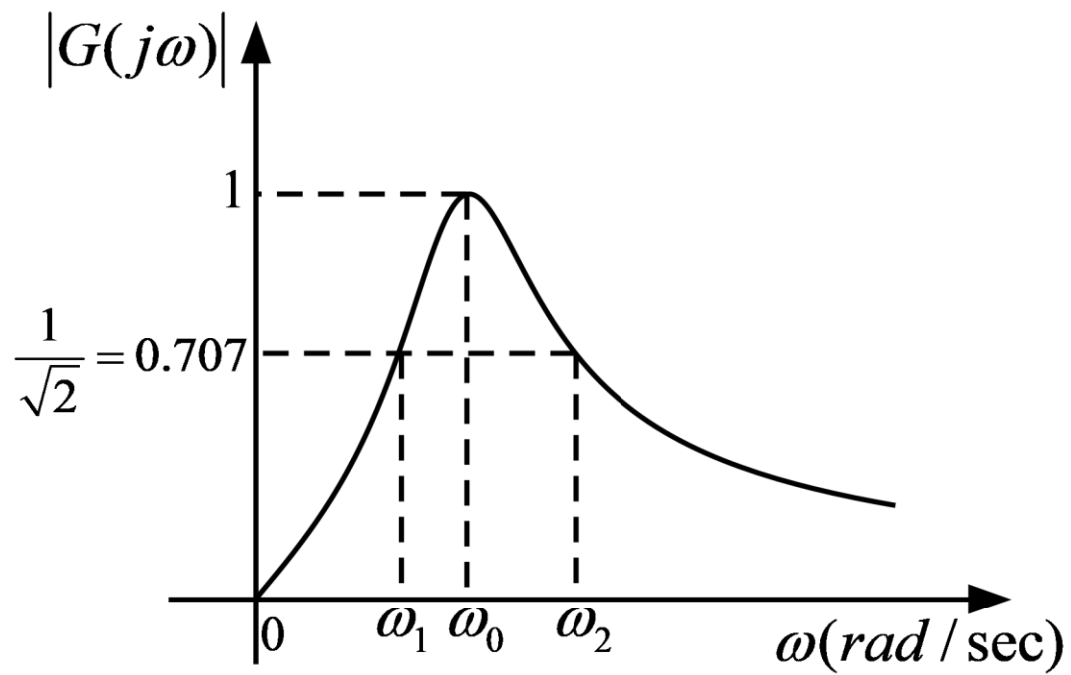
RLC Resonant Circuits

Series Resonant Circuits



$$V(j\omega) = \frac{R}{j\omega L + R + \frac{1}{j\omega C}} E(j\omega) = \frac{j\omega RC}{(j\omega)^2 LC + (j\omega)RC + 1} E(j\omega)$$

$$G(j\omega) = \frac{j\omega RC}{(j\omega)^2 LC + (j\omega)RC + 1} = \frac{j\omega \frac{R}{L}}{(j\omega)^2 + (j\omega) \frac{R}{L} + \frac{1}{LC}}$$



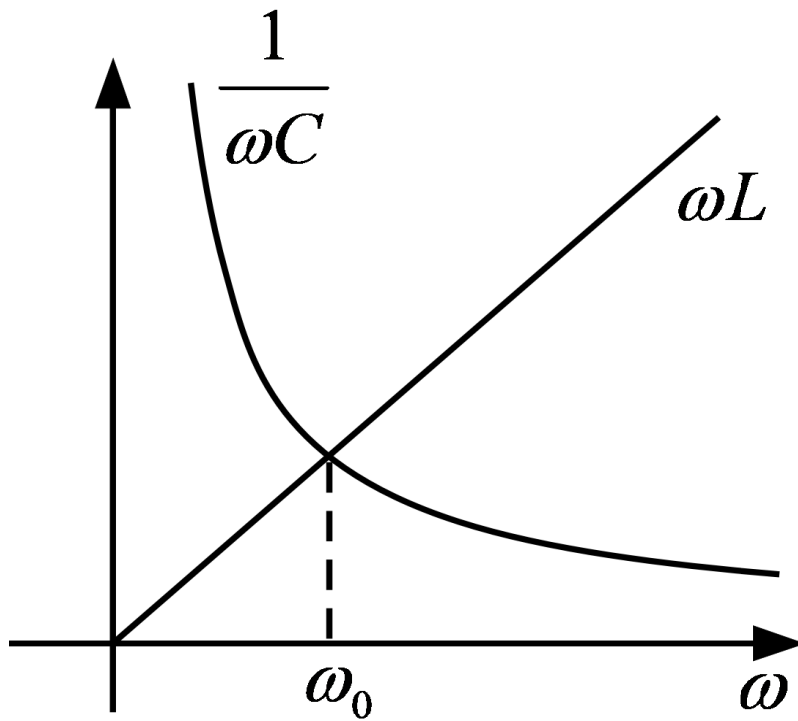
$$Z(j\omega) = j\omega L + R + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$|Z(j\omega)| = \left| R + j\left(\omega L - \frac{1}{\omega C}\right) \right| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$V(j\omega) = \frac{R}{Z(j\omega)} E(j\omega), G(j\omega) = \frac{V(j\omega)}{E(j\omega)} = \frac{R}{Z(j\omega)}$$

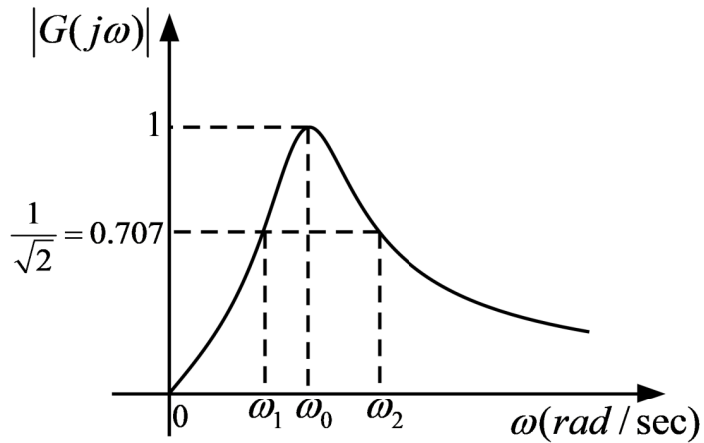
$$|G(j\omega)| = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\omega L = \frac{1}{\omega C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} : |G(j\omega)| = 1$$



$$\omega L = \frac{1}{\omega C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} : |G(j\omega)| = 1$$

Band Width, Quality Factor



$$B = \omega_2 - \omega_1$$

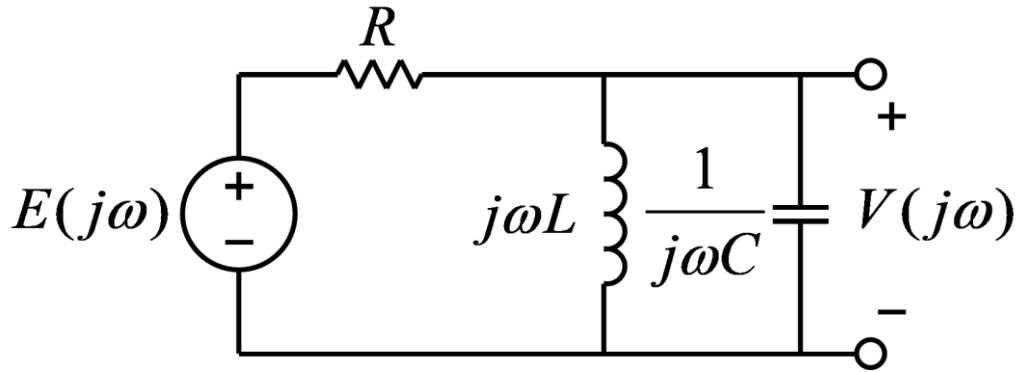
$$G(j\omega) = \frac{(j\omega) \left(\frac{\omega_0}{Q_0} \right)}{(j\omega)^2 + (j\omega) \left(\frac{\omega_0}{Q_0} \right) + \omega_0^2}$$

$$G(j\omega) = \frac{j\omega \frac{R}{L}}{(j\omega)^2 + (j\omega) \frac{R}{L} + \frac{1}{LC}}$$

$$Q_0 = \frac{L\omega_0}{R} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

$$Q_0 = \frac{\omega_0}{B} = \frac{\omega_0}{\omega_2 - \omega_1}$$

Parallel Resonant Circuits

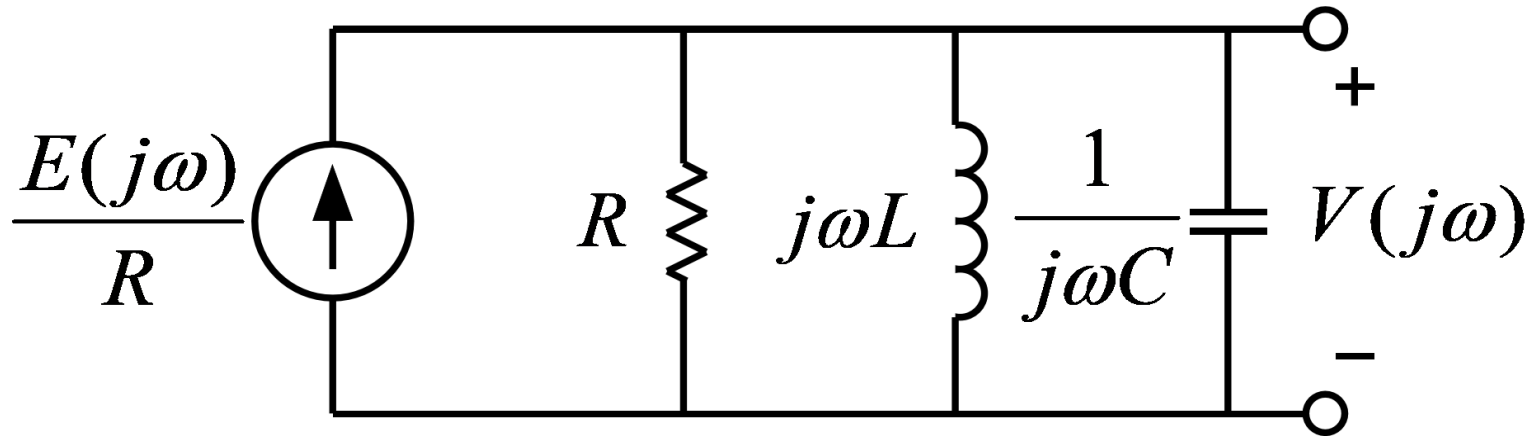


$$j\omega L \parallel \frac{1}{j\omega C} = \frac{\frac{j\omega L}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega L}{(j\omega)^2 LC + 1}$$

$$V(j\omega) = \frac{\frac{j\omega L}{(j\omega)^2 LC + 1}}{R + \frac{j\omega L}{(j\omega)^2 LC + 1}} E(j\omega) = \frac{j\omega L}{(j\omega)^2 LCR + (j\omega)L + R} E(j\omega)$$

$$G(j\omega) = \frac{j\omega \left(\frac{L}{R} \right)}{(j\omega)^2 LC + (j\omega) \left(\frac{L}{R} \right) + 1} = \frac{j\omega \left(\frac{1}{RC} \right)}{(j\omega)^2 + (j\omega) \left(\frac{1}{RC} \right) + \frac{1}{LC}}$$

Admittance



$$Y(j\omega) = \frac{1}{R} + j\omega C + \frac{1}{j\omega L} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

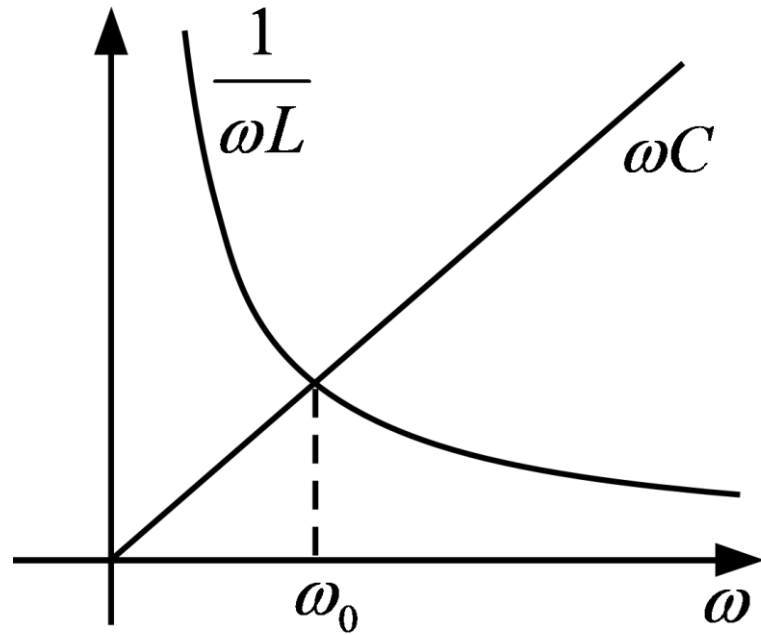
$$Y(j\omega) = \frac{1}{R} + j\omega C + \frac{1}{j\omega L} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

$$|Y(j\omega)| = \left| \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right) \right| = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

$$V(j\omega) = \frac{1}{Y(j\omega)} \frac{E(j\omega)}{R}$$

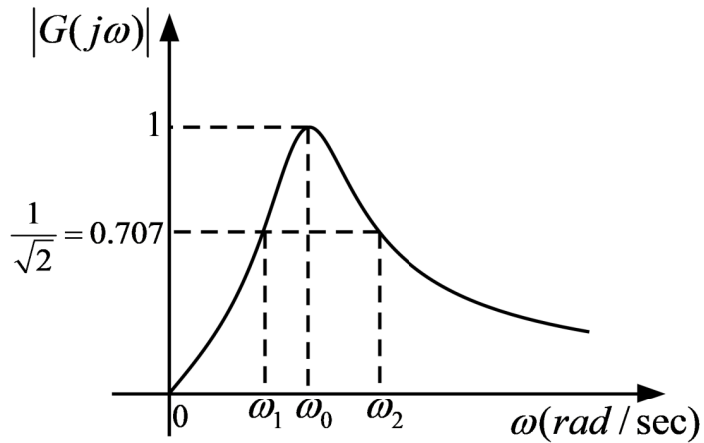
$$G(j\omega) = \frac{V(j\omega)}{E(j\omega)} = \frac{1}{R} \frac{1}{Y(j\omega)}$$

$$|G(j\omega)| = \frac{1}{R} \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$



$$\omega L = \frac{1}{\omega C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} : |G(j\omega)| = 1$$

Band Width, Quality Factor



$$B = \omega_2 - \omega_1$$

$$G(j\omega) = \frac{(j\omega) \left(\frac{\omega_0}{Q_0} \right)}{(j\omega)^2 + (j\omega) \left(\frac{\omega_0}{Q_0} \right) + \omega_0^2}$$

$$G(j\omega) = \frac{j\omega \left(\frac{1}{RC} \right)}{(j\omega)^2 + (j\omega) \left(\frac{1}{RC} \right) + \frac{1}{LC}}$$

$$Q_0 = RC\omega_0 = R\sqrt{\frac{C}{L}}$$

$$Q_0 = \frac{\omega_0}{B} = \frac{\omega_0}{\omega_2 - \omega_1}$$