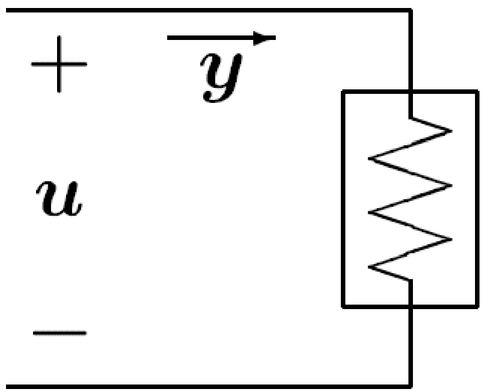
Chapter 6
Passivity

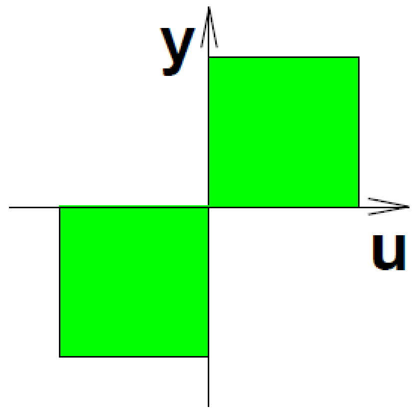
Passive & Stability

Positive Real Transfer Functions

Memoryless Functions



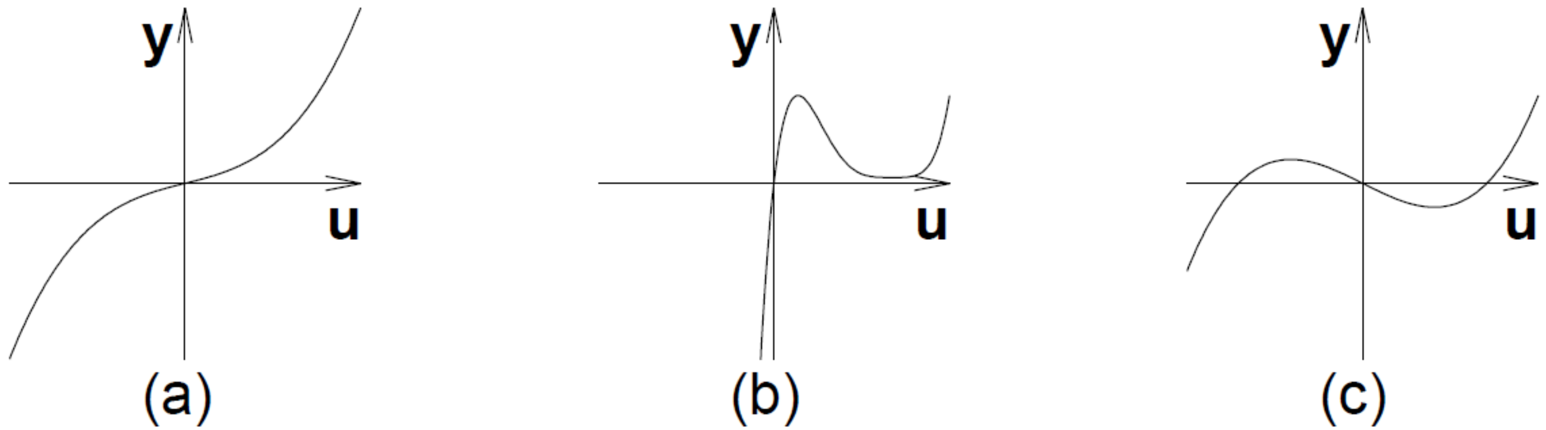
(a)



(b)

$$\text{power inflow} = uy$$

Resistor is passive if $uy \geq 0$



Passive　　Passive　　Not passive

$$y = h(t, u), \quad h \in [0, \infty]$$

Vector case:

$$y = h(t, u), \quad h^T = \left[\begin{array}{cccc} h_1, & h_2, & \cdots, & h_p \end{array} \right]$$

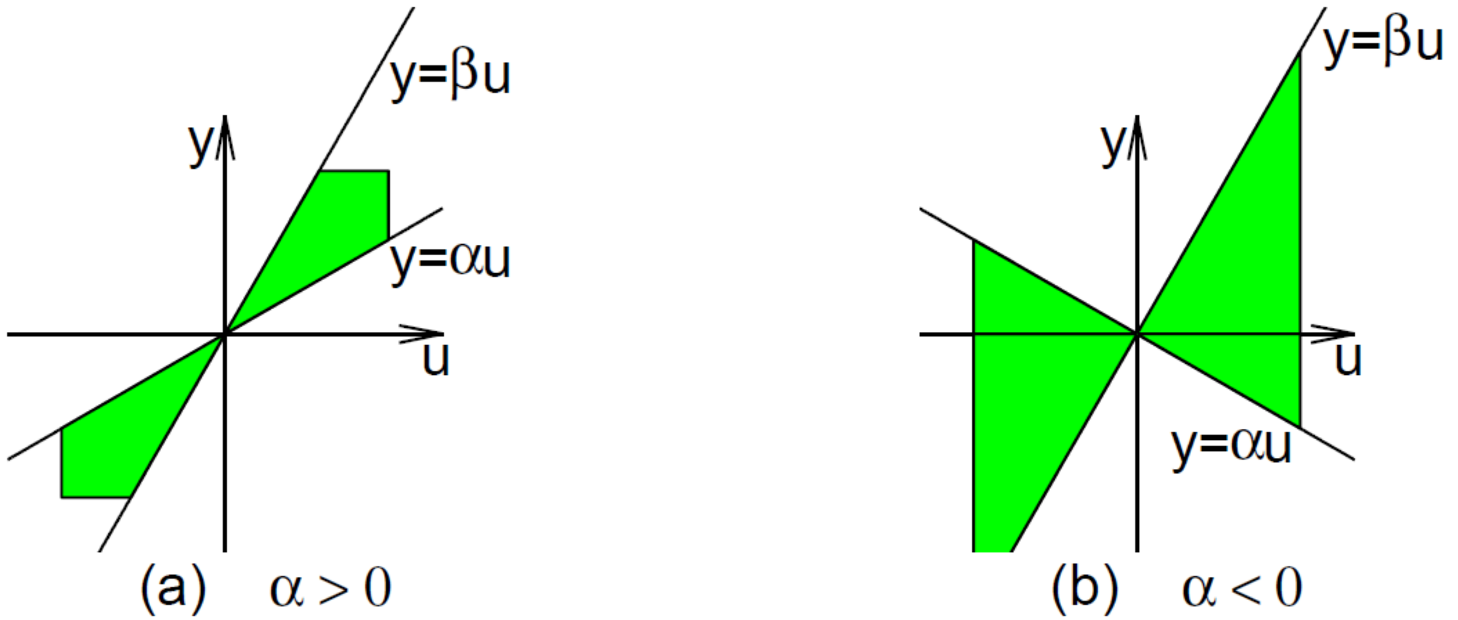
power inflow $= \Sigma_{i=1}^{p} u_i y_i = u^T y$

Definition: $y = h(t, u)$ is

- passive if $u^T y \geq 0$
- lossless if $u^T y = 0$
- input strictly passive if $u^T y \geq u^T \varphi(u)$ for some function φ where $u^T \varphi(u) > 0$, $\forall\, u \neq 0$
- output strictly passive if $u^T y \geq y^T \rho(y)$ for some function ρ where $y^T \rho(y) > 0$, $\forall\, y \neq 0$

Sector Nonlinearity: h belongs to the sector $[\alpha, \beta]$ ($h \in [\alpha, \beta]$) if

$$\alpha u^2 \leq u h(t, u) \leq \beta u^2$$



Also, $h \in (\alpha, \beta]$, $h \in [\alpha, \beta)$, $h \in (\alpha, \beta)$

$$\alpha u^2 \le uh(t,u) \le \beta u^2 \Leftrightarrow [h(t,u) - \alpha u][h(t,u) - \beta u] \le 0$$

Definition: A memoryless function $h(t,u)$ is said to belong to the sector

- $[0, \infty]$ if $u^T h(t,u) \ge 0$
- $[K_1, \infty]$ if $u^T [h(t,u) - K_1 u] \ge 0$
- $[0, K_2]$ with $K_2 = K_2^T > 0$ if
  $h^T(t,u)[h(t,u) - K_2 u] \le 0$
- $[K_1, K_2]$ with $K = K_2 - K_1 = K^T > 0$ if

$$[h(t,u) - K_1 u]^T [h(t,u) - K_2 u] \le 0$$

Example

$$\boldsymbol{h(u)} = \begin{bmatrix} \boldsymbol{h_1(u_1)} \\ \boldsymbol{h_2(u_2)} \end{bmatrix}, \quad \boldsymbol{h_i \in [\alpha_i, \beta_i],\ \ \beta_i > \alpha_i\ \ i = 1, 2}$$

$$\boldsymbol{K_1} = \begin{bmatrix} \boldsymbol{\alpha_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\alpha_2} \end{bmatrix}, \quad \boldsymbol{K_2} = \begin{bmatrix} \boldsymbol{\beta_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\beta_2} \end{bmatrix}$$

$$\boldsymbol{h \in [K_1, K_2]}$$

$$\boldsymbol{K = K_2 - K_1} = \begin{bmatrix} \boldsymbol{\beta_1 - \alpha_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\beta_2 - \alpha_2} \end{bmatrix}$$

Example

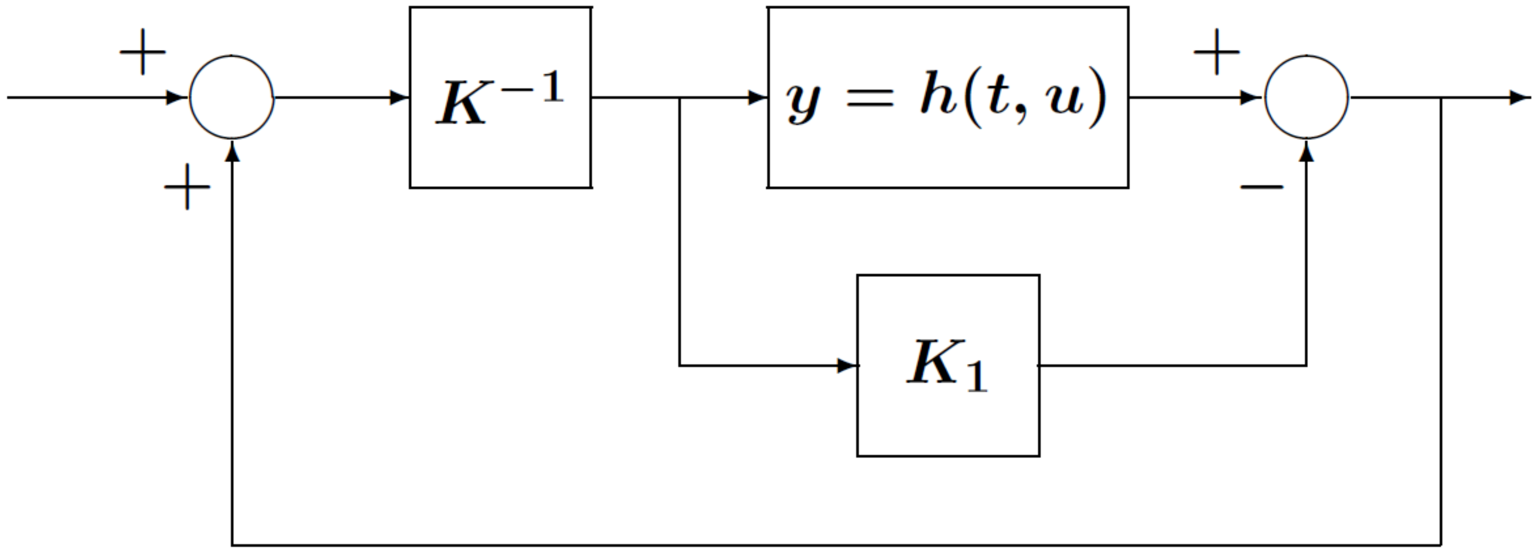
$$\|h(u) - Lu\| \le \gamma \|u\|$$

$$K_1 = L - \gamma I, \quad K_2 = L + \gamma I$$

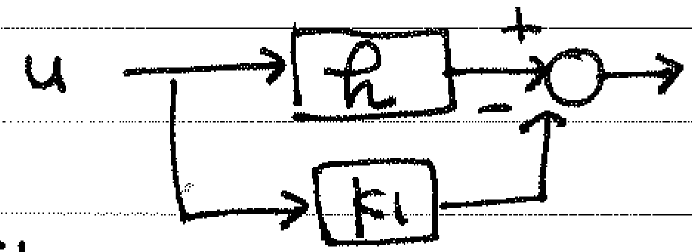
$$[h(u) - K_1 u]^T [h(u) - K_2 u] = \|h(u) - Lu\|^2 - \gamma^2 \|u\|^2 \le 0$$

$$K = K_2 - K_1 = 2\gamma I$$

A function in the sector $[K_1, K_2]$ can be transformed into a function in the sector $[0, \infty]$ by input feedforward followed by output feedback



$$[K_1, K_2] \xrightarrow{\text{Feedforward}} [0, K] \xrightarrow{K^{-1}} [0, I] \xrightarrow{\text{Feedback}} [0, \infty]$$



$h \in [k_1, k_2]$

$$[h - k_1 u]^T [h - k_2 u] \leq 0$$

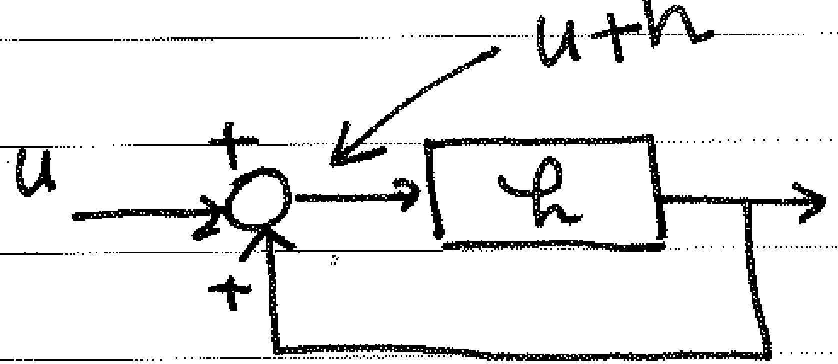
$$h_1 = h - k_1 u$$

$$h_1^T [\underbrace{h - k_1 u}_{h_1} + k_1 u - k_2 u] \leq 0$$

$$h_1^T [h_1 - k u] \leq 0$$

$$k = k_2 - k_1$$

$$h_1 \in [0, k]$$



$$h^T[h-(u+h)] \leq 0$$

$$h^T u \geq 0$$

$$\rightarrow \quad \in [0, \infty]$$

State Models

Let us now define passivity for a dynamical system represented by the state model

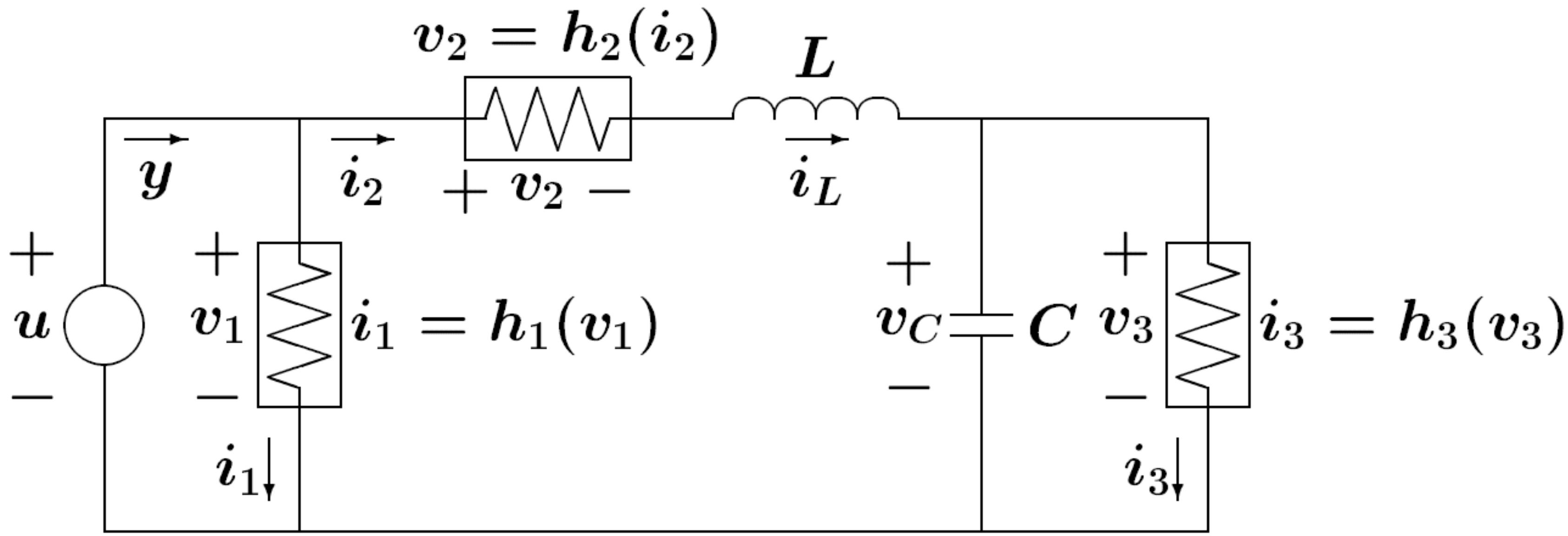
$$\dot{x} = f(x,u) \tag{6.6}$$

$$y = h(x,u) \tag{6.7}$$

where $f : R^n \times R^p \to R^n$ is locally Lipschitz, $h : R^n \times R^p \to R^p$ is continuous, $f(0,0) = 0$, and $h(0,0) = 0$. The system has the same number of inputs and outputs. The following RLC circuit motivates the definition.

- The origin is an equilibrium point.

State Models



$$
\begin{aligned}
L\dot{x}_1 &= u - h_2(x_1) - x_2 \\
C\dot{x}_2 &= x_1 - h_3(x_2) \\
y &= x_1 + h_1(u)
\end{aligned}
$$

$$V(x) = \tfrac{1}{2}Lx_1^2 + \tfrac{1}{2}Cx_2^2$$

$$\int_0^t u(s)y(s)\,ds \geq V(x(t)) - V(x(0))$$

$$u(t)y(t) \geq \dot{V}(x(t), u(t))$$

$$\begin{aligned}
\dot{V} &= Lx_1\dot{x}_1 + Cx_2\dot{x}_2 \\
&= x_1[u - h_2(x_1) - x_2] + x_2[x_1 - h_3(x_2)] \\
&= x_1[u - h_2(x_1)] - x_2h_3(x_2) \\
&= [x_1 + h_1(u)]u - uh_1(u) - x_1h_2(x_1) - x_2h_3(x_2) \\
&= uy - uh_1(u) - x_1h_2(x_1) - x_2h_3(x_2)
\end{aligned}$$

$$uy = \dot{V} + uh_1(u) + x_1h_2(x_1) + x_2h_3(x_2)$$

If h_1, h_2, and h_3 are passive, $uy \geq \dot{V}$ and the system is passive

Case 1: If $h_1 = h_2 = h_3 = 0$, then $uy = \dot{V}$; no energy dissipation; the system is lossless

Case 2: If $h_1 \in (0, \infty]$ $(uh_1(u) > 0$ for $u \neq 0)$, then

$h_2, h_3 \in [0, \infty]$

$$uy \geq \dot{V} + uh_1(u)$$

The energy absorbed over $[0, t]$ will be greater than the increase in the stored energy, unless the input $u(t)$ is identically zero. This is a case of input strict passivity

Case 3: If $h_1 = 0$ and $h_3 \in [0, \infty]$, then

$$y = x_1 \text{ and } uy \geq \dot{V} + y h_2(y)$$

The energy absorbed over $[0, t]$ will be greater than the increase in the stored energy, unless the output y is identically zero. This is a case of output strict passivity

Case 4: If $h_2 \in (0, \infty)$ and $h_3 \in (0, \infty)$, then

$$uy \geq \dot{V} + x_1 h_2(x_1) + x_2 h_3(x_2)$$

$x_1 h_2(x_1) + x_2 h_3(x_2)$ is a positive definite function of x. This is a case of state strict passivity because the energy absorbed over $[0, t]$ will be greater than the increase in the stored energy, unless the state x is identically zero

Definition: The system

$$\dot{x} = f(x, u), \qquad y = h(x, u)$$

is passive if there is a continuously differentiable positive semidefinite function $V(x)$ (the storage function) such that

$$u^T y \geq \dot{V} = \frac{\partial V}{\partial x} f(x, u), \ \ \forall \ (x, u)$$

Moreover, it is said to be

- lossless if $u^T y = \dot{V}$
- input strictly passive if $u^T y \geq \dot{V} + u^T \varphi(u)$ for some function φ such that $u^T \varphi(u) > 0, \ \forall \ u \neq 0$

- output strictly passive if $u^T y \geq \dot{V} + y^T \rho(y)$ for some function ρ such that $y^T \rho(y) > 0, \; \forall \, y \neq 0$

- strictly passive if $u^T y \geq \dot{V} + \psi(x)$ for some positive definite function ψ

Example

$$\dot{x} = u, \qquad y = x$$

$$V(x) = \frac{1}{2}x^2 \;\Rightarrow\; uy = \dot{V} \;\Rightarrow\; \text{Lossless}$$

Example

$$\dot{x} = u, \qquad y = x + h(u), \quad h \in [0, \infty]$$

$$V(x) = \tfrac{1}{2}x^2 \;\Rightarrow\; uy = \dot{V} + uh(u) \;\Rightarrow\; \text{Passive}$$

$$h \in (0, \infty] \;\Rightarrow\; uh(u) > 0 \;\forall\; u \neq 0$$

$$\Rightarrow \; \text{Input strictly passive}$$

Example

$$\dot{x} = -h(x) + u, \qquad y = x, \quad h \in [0, \infty]$$

$$V(x) = \tfrac{1}{2}x^2 \;\Rightarrow\; uy = \dot{V} + yh(y) \;\Rightarrow\; \text{Passive}$$

$$h \in (0, \infty] \;\Rightarrow\; \text{Output strictly passive}$$

Example

$$\dot{x} = u, \qquad y = h(x), \quad h \in [0, \infty]$$

$$V(x) = \int_0^x h(\sigma)\, d\sigma \;\Rightarrow\; \dot{V} = h(x)\dot{x} = yu \;\Rightarrow\; \text{Lossless}$$

Example

$$a\dot{x} = -x + u, \qquad y = h(x), \quad h \in [0, \infty]$$

$$V(x) = a\int_0^x h(\sigma)\, d\sigma \;\Rightarrow\; \dot{V} = h(x)(-x+u) = yu - xh(x)$$

$$yu = \dot{V} + xh(x) \;\Rightarrow\; \text{Passive}$$

$$h \in (0, \infty] \;\Rightarrow\; \text{Strictly passive}$$

Positive Linear Systems

- PR and SPR Transfer Functions

$$h(p) = \frac{b_m p^m + b_{m-1} p^{m-1} + \dots + b_o}{p^n + a_{n-1} p^{n-1} + \dots + a_o}$$

The coefficients of the numerator and denominator polynomials are assumed to be real numbers and $n \geq m$. The difference $n - m$ between the order of the denominator and that of the numerator is called the *relative degree* of the system.

Definition 4.10 *A transfer function $h(p)$ is positive real if*

$$Re[h(p)] \geq 0 \quad \text{for all} \quad Re[p] \geq 0 \tag{4.33}$$

It is strictly positive real if $h(p-\varepsilon)$ is positive real for some $\varepsilon > 0$.

Example

Example 4.14: A strictly positive real function

Consider the rational function

$$h(p) = \frac{1}{p + \lambda}$$

which is the transfer function of a first-order system, with $\lambda > 0$. Corresponding to the complex variable $p = \sigma + j\omega$,

$$h(p) = \frac{1}{(\sigma + \lambda) + j\omega} = \frac{\sigma + \lambda - j\omega}{(\sigma + \lambda)^2 + \omega^2}$$

Obviously, $\text{Re}[h(p)] \geq 0$ if $\sigma \geq 0$. Thus, $h(p)$ is a positive real function. In fact, one can easily see that $h(p)$ is strictly positive real, for example by choosing $\varepsilon = \lambda/2$ in Definition 4.9. □

Theorem

Theorem 4.10 *A transfer function $h(p)$ is strictly positive real (SPR) if and only if*

i) $h(p)$ is a strictly stable transfer function

ii) the real part of $h(p)$ is strictly positive along the $j\omega$ axis, i.e.,

$$\forall\ \omega \geq 0 \quad \mathrm{Re}[h(j\omega)] > 0 \tag{4.34}$$

Theorem

The above theorem implies simple *necessary* conditions for asserting whether a given transfer function $h(p)$ is SPR:

- $h(p)$ is strictly stable

- The Nyquist plot of $h(j\omega)$ lies entirely in the right half complex plane. Equivalently, the phase shift of the system in response to sinusoidal inputs is always less than 90^{o}

- $h(p)$ has relative degree is 0 or 1

- $h(p)$ is strictly minimum-phase (*i.e.*, all its zeros are strictly in the left-half plane)

Example

Example 4.15: SPR and non-SPR transfer functions

Consider the following systems

$$h_1(p) = \frac{p-1}{p^2 + a\,p + b}$$

$$h_2(p) = \frac{p+1}{p^2 - p + 1}$$

$$h_3(p) = \frac{1}{p^2 + a\,p + b}$$

$$h_4(p) = \frac{p+1}{p^2 + p + 1}$$

Example

The transfer functions h_1, h_2 , and h_3 are not SPR, because h_1 is non-minimum phase, h_2 is unstable, and h_3 has relative degree larger than 1.

Is the (strictly stable, minimum-phase, and of relative degree 1) function h_4 actually SPR? We have

$$h_4(j\omega) = \frac{j\omega + 1}{-\omega^2 + j\omega + 1} = \frac{[j\omega + 1]\,[-\omega^2 - j\omega + 1]}{[1 - \omega^2]^2 + \omega^2}$$

(where the second equality is obtained by multiplying numerator and denominator by the complex conjugate of the denominator) and thus

$$\mathrm{Re}[\ h_4(j\omega)\ ] = \frac{-\omega^2 + 1 + \omega^2}{[1 - \omega^2]^2 + \omega^2} = \frac{1}{[1 - \omega^2]^2 + \omega^2}$$

which shows that h_4 is SPR (since it is also strictly stable). Of course, condition (4.34) can also be checked directly on a computer. □

Example

Example 4.16: Consider the transfer function of an integrator,

$$h(p) = \frac{1}{p}$$

Its value corresponding to $p = \sigma + j\omega$ is

$$h(p) = \frac{\sigma - j\omega}{\sigma^2 + \omega^2}$$

One easily sees from Definition 4.9 that $h(p)$ is PR but not SPR. □

Theorem

Theorem 4.11 *A transfer function $h(p)$ is positive real if, and only if,*

i) $h(p)$ is a stable transfer function

(ii) The poles of $h(p)$ on the $j\omega$ axis are simple (i.e., distinct) and the associated residues are real and non-negative

iii) $Re[h(j\omega)] \geq 0$ for any $\omega \geq 0$ such that $j\omega$ is not a pole of $h(p)$

Definition: A $p \times p$ proper rational transfer function matrix $G(s)$ is positive real if

- poles of all elements of $G(s)$ are in $Re[s] \leq 0$
- for all real ω for which $j\omega$ is not a pole of any element of $G(s)$, the matrix $G(j\omega) + G^T(-j\omega)$ is positive semidefinite
- any pure imaginary pole $j\omega$ of any element of $G(s)$ is a simple pole and the residue matrix $\lim_{s \to j\omega}(s - j\omega)G(s)$ is positive semidefinite Hermitian

$G(s)$ is called strictly positive real if $G(s - \varepsilon)$ is positive real for some $\varepsilon > 0$

Scalar Case ($p = 1$):

$$G(j\omega) + G^T(-j\omega) = 2Re[G(j\omega)]$$

$Re[G(j\omega)]$ is an even function of ω. The second condition of the definition reduces to

$$Re[G(j\omega)] \geq 0, \; \forall \; \omega \in [0, \infty)$$

which holds when the Nyquist plot of of $G(j\omega)$ lies in the closed right-half complex plane

This is true only if the relative degree of the transfer function is zero or one

Lemma: Suppose $\det\left[G(s) + G^T(-s)\right]$ is not identically zero. Then, $G(s)$ is strictly positive real if and only if

- $G(s)$ is Hurwitz
- $G(j\omega) + G^T(-j\omega) > 0, \ \forall\, \omega \in R$
- $G(\infty) + G^T(\infty) > 0$ or

$$\lim_{\omega\to\infty} \omega^2 M^T[G(j\omega) + G^T(-j\omega)]M > 0$$

for any $p \times (p-q)$ full-rank matrix M such that

$$M^T[G(\infty) + G^T(\infty)]M = 0$$

$q = \mathrm{rank}[G(\infty) + G^T(\infty)]$

Scalar Case ($p = 1$): $G(s)$ is strictly positive real if and only if

- $G(s)$ is Hurwitz
- $Re[G(j\omega)] > 0, \ \forall \ \omega \in [0, \infty)$
- $G(\infty) > 0$ or

$$G(\infty) = 0 \qquad \lim_{\omega \to \infty} \omega^2 Re[G(j\omega)] > 0$$

Example:

$$G(s) = \frac{1}{s}$$

has a simple pole at $s = 0$ whose residue is 1

$$Re[G(j\omega)] = Re\left[\frac{1}{j\omega}\right] = 0, \;\; \forall\, \omega \neq 0$$

Hence, G is positive real. It is not strictly positive real since

$$\frac{1}{(s - \varepsilon)}$$

has a pole in $Re[s] > 0$ for any $\varepsilon > 0$

Example:

$$G(s) = \frac{1}{s+a}, \; a > 0, \quad \text{is Hurwitz}$$

$$Re[G(j\omega)] = \frac{a}{\omega^2 + a^2} > 0, \quad \forall\, \omega \in [0, \infty)$$

$$\lim_{\omega\to\infty} \omega^2 Re[G(j\omega)] = \lim_{\omega\to\infty} \frac{\omega^2 a}{\omega^2 + a^2} = a > 0 \quad \Rightarrow \quad \text{G is SPR}$$

Example:

$$G(s) = \frac{1}{s^2 + s + 1}, \quad Re[G(j\omega)] = \frac{1-\omega^2}{(1-\omega^2)^2 + \omega^2}$$

G is not PR

Example:

$$G(s) = \begin{bmatrix} \frac{s+2}{s+1} & \frac{1}{s+2} \\ \\ \frac{-1}{s+2} & \frac{2}{s+1} \end{bmatrix} \text{ is Hurwitz}$$

$$G(j\omega) + G^T(-j\omega) = \begin{bmatrix} \frac{2(2+\omega^2)}{1+\omega^2} & \frac{-2j\omega}{4+\omega^2} \\ \\ \frac{2j\omega}{4+\omega^2} & \frac{4}{1+\omega^2} \end{bmatrix} > 0, \quad \forall\, \omega \in R$$

$$G(\infty) + G^T(\infty) = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, \quad M = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\lim_{\omega \to \infty} \omega^2 M^T [G(j\omega) + G^T(-j\omega)] M = 4 \quad \Rightarrow \quad G \text{ is SPR}$$

Positive Real Lemma: Let

$$G(s) = C(sI - A)^{-1}B + D$$

where (A, B) is controllable and (A, C) is observable. $G(s)$ is positive real if and only if there exist matrices $P = P^T > 0$, L, and W such that

$$\begin{aligned} PA + A^TP &= -L^TL \\ PB &= C^T - L^TW \\ W^TW &= D + D^T \end{aligned}$$

Kalman–Yakubovich–Popov Lemma: Let

$$G(s) = C(sI - A)^{-1}B + D$$

where (A, B) is controllable and (A, C) is observable. $G(s)$ is strictly positive real if and only if there exist matrices $P = P^T > 0$, L, and W, and a positive constant ε such that

$$\begin{aligned} PA + A^T P &= -L^T L - \varepsilon P \\ PB &= C^T - L^T W \\ W^T W &= D + D^T \end{aligned}$$

Lemma: The linear time-invariant minimal realization

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

with

$$G(s) = C(sI - A)^{-1}B + D$$

is

- passive if $G(s)$ is positive real
- strictly passive if $G(s)$ is strictly positive real

Proof: Apply the PR and KYP Lemmas, respectively, and use $V(x) = \frac{1}{2}x^T P x$ as the storage function

$$
\begin{aligned}
u^T y &- \frac{\partial V}{\partial x}(Ax + Bu) \\
&= u^T(Cx + Du) - x^T P(Ax + Bu) \\
&= u^T Cx + \tfrac{1}{2}u^T(D + D^T)u \\
&\quad - \tfrac{1}{2}x^T(PA + A^T P)x - x^T PBu \\
&= u^T(B^T P + W^T L)x + \tfrac{1}{2}u^T W^T W u \\
&\quad + \tfrac{1}{2}x^T L^T Lx + \tfrac{1}{2}\varepsilon x^T Px - x^T PBu \\
&= \tfrac{1}{2}(Lx + Wu)^T(Lx + Wu) + \tfrac{1}{2}\varepsilon x^T Px \ \geq \tfrac{1}{2}\varepsilon x^T Px
\end{aligned}
$$

In the case of the PR Lemma, $\varepsilon = 0$, and we conclude that the system is passive; in the case of the KYP Lemma, $\varepsilon > 0$, and we conclude that the system is strictly passive

Connection with Lyapunov Stability

Lemma: If the system

$$\dot{x} = f(x, u), \qquad y = h(x, u)$$

is passive with a positive definite storage function $V(x)$, then the origin of $\dot{x} = f(x, 0)$ is stable

Proof:

$$u^T y \geq \frac{\partial V}{\partial x} f(x, u) \;\Rightarrow\; \frac{\partial V}{\partial x} f(x, 0) \leq 0$$

Lemma: If the system

$$\dot{x} = f(x, u), \qquad y = h(x, u)$$

is strictly passive, then the origin of $\dot{x} = f(x, 0)$ is asymptotically stable. Furthermore, if the storage function is radially unbounded, the origin will be globally asymptotically stable

Proof: The storage function $V(x)$ is positive definite

$$u^T y \geq \frac{\partial V}{\partial x} f(x, u) + \psi(x) \;\; \Rightarrow \;\; \frac{\partial V}{\partial x} f(x, 0) \leq -\psi(x)$$

Why is $V(x)$ positive definite? Let $\phi(t; x)$ be the solution of $\dot{z} = f(z, 0), \; z(0) = x$

$$\dot{V} \leq -\psi(x)$$

$$V(\phi(\tau, x)) - V(x) \leq -\int_0^{\tau} \psi(\phi(t; x))\, dt, \;\; \forall\, \tau \in [0, \delta]$$

$$V(\phi(\tau, x)) \geq 0 \;\Rightarrow\; V(x) \geq \int_0^{\tau} \psi(\phi(t; x))\, dt$$

$$V(\bar{x}) = 0 \;\Rightarrow\; \int_0^{\tau} \psi(\phi(t; \bar{x}))\, dt = 0, \; \forall\, \tau \in [0, \delta]$$

$$\Rightarrow\; \psi(\phi(t; \bar{x})) \equiv 0 \;\Rightarrow\; \phi(t; \bar{x}) \equiv 0 \;\Rightarrow\; \bar{x} = 0$$

Definition: The system

$$\dot{x} = f(x, u), \qquad y = h(x, u)$$

is zero-state observable if no solution of $\dot{x} = f(x, 0)$ can stay identically in $S = \{h(x, 0) = 0\}$, other than the zero solution $x(t) \equiv 0$

Linear Systems

$$\dot{x} = Ax, \quad y = Cx$$

Observability of (A, C) is equivalent to

$$y(t) = Ce^{At}x(0) \equiv 0 \;\Leftrightarrow\; x(0) = 0 \;\Leftrightarrow\; x(t) \equiv 0$$

Lemma: If the system

$$\dot{x} = f(x, u), \qquad y = h(x, u)$$

is output strictly passive and zero-state observable, then the origin of $\dot{x} = f(x, 0)$ is asymptotically stable. Furthermore, if the storage function is radially unbounded, the origin will be globally asymptotically stable

Proof: The storage function $V(x)$ is positive definite

$$u^T y \geq \frac{\partial V}{\partial x} f(x, u) + y^T \rho(y) \;\Rightarrow\; \frac{\partial V}{\partial x} f(x, 0) \leq -y^T \rho(y)$$

$$\dot{V}(x(t)) \equiv 0 \;\Rightarrow\; y(t) \equiv 0 \;\Rightarrow x(t) \equiv 0$$

Apply the invariance principle

Example

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -ax_1^3 - kx_2 + u, \quad y = x_2, \quad a, k > 0$$

$$V(x) = \tfrac{1}{4}ax_1^4 + \tfrac{1}{2}x_2^2$$

$$\dot{V} = ax_1^3 x_2 + x_2(-ax_1^3 - kx_2 + u) = -ky^2 + yu$$

The system is output strictly passive

$$y(t) \equiv 0 \;\Leftrightarrow\; x_2(t) \equiv 0 \;\Rightarrow\; ax_1^3(t) \equiv 0 \;\Rightarrow\; x_1(t) \equiv 0$$

The system is zero-state observable. V is radially unbounded. Hence, the origin of the unforced system is globally asymptotically stable