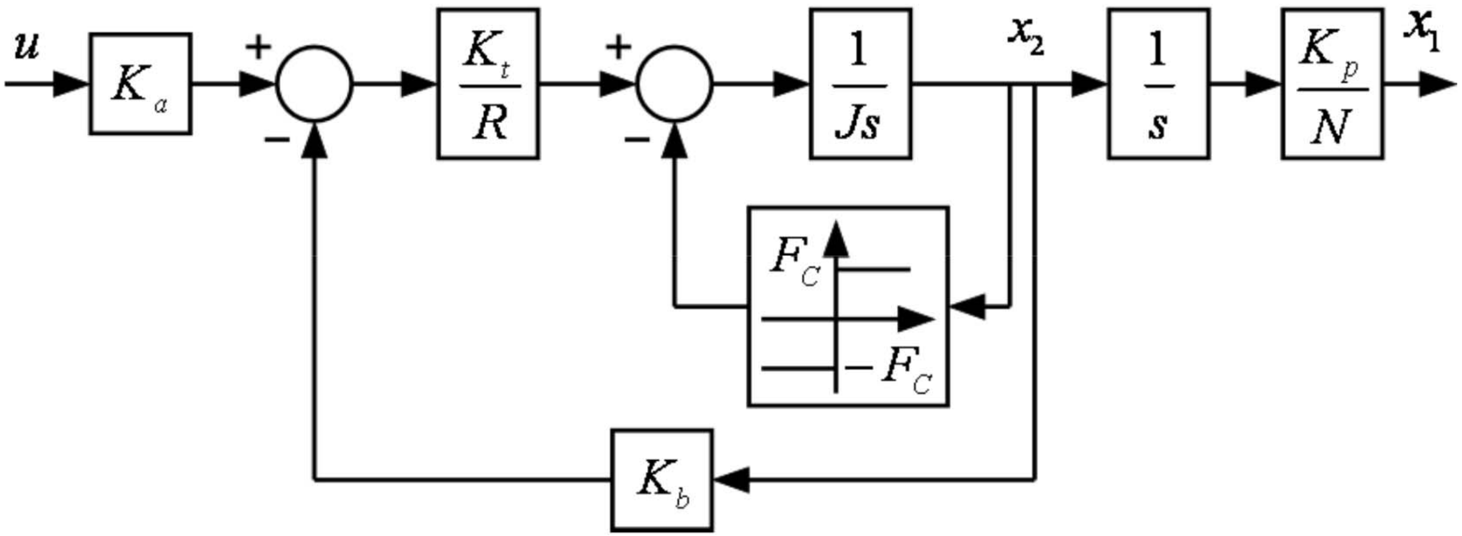
Topics on Labs

- Limit Cycle in Motor Control System
- Magnetic Levitation
- Ball & Beam

Limit Cycle in Motor Control System

Motor



$$\dot{x}_1 = \frac{K_p}{N} x_2$$

$$\dot{x}_2 = \frac{K_t}{R}\frac{1}{J}\left(K_a u - K_b x_2\right) = -\frac{K_t K_b}{RJ} x_2 + \frac{K_t K_a}{RJ} u$$

Motor

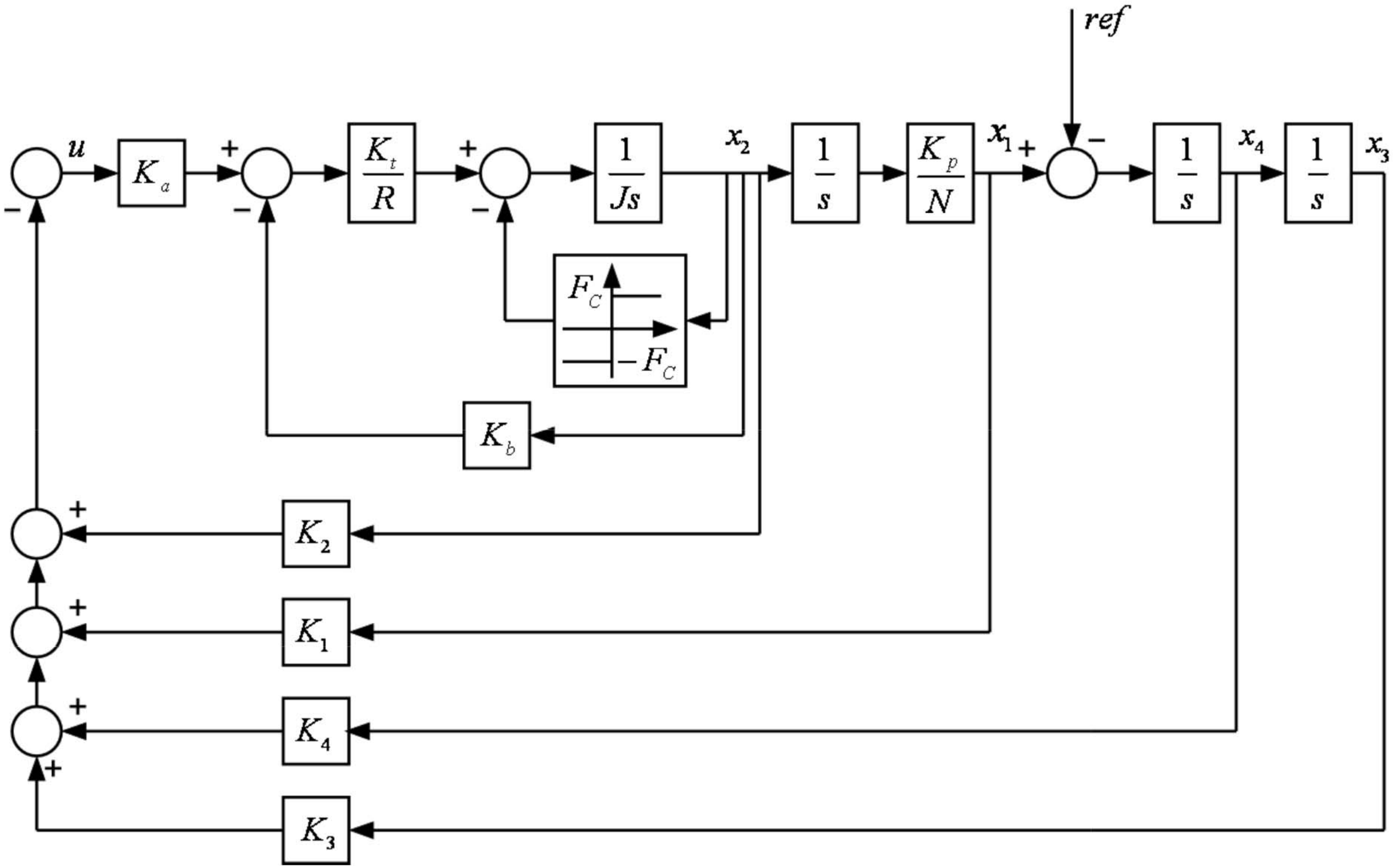
$$\dot{x} = Ax + Bu$$

$$y = Cx$$

$$A = \begin{bmatrix} 0 & \frac{K_p}{N} \\ 0 & -\frac{K_t K_b}{RJ} \end{bmatrix}, B = \begin{bmatrix} 0 \\ \frac{K_t K_a}{RJ} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Tracking Controller



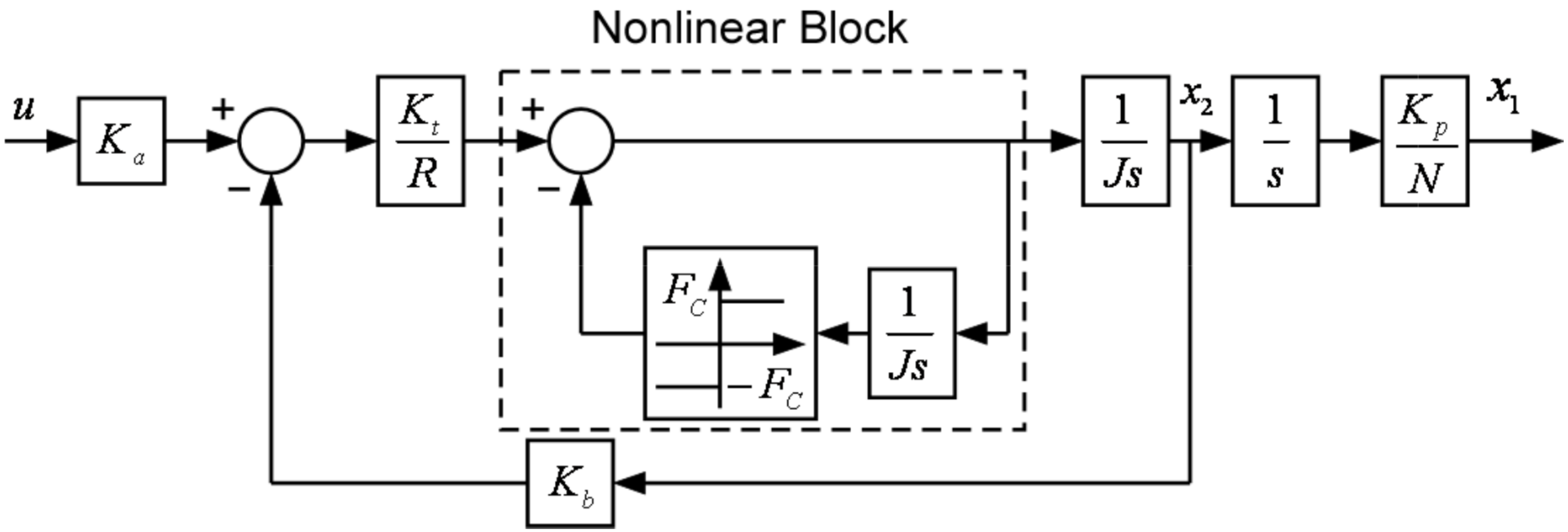
Tracking Controller

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} A & & 0 & 0 \\ & & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} B \\ \\ 0 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} ref$$

$$u = -(K_1 x_1 + K_2 x_2 + K_3 x_3 + K_4 x_4)$$

$$J = \int_0^\infty \left(\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix} Q \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + Ru^2 \right) dt$$

Motor



$$F_C = (0.5A) \times K_t = 0.0333$$

Describing Function

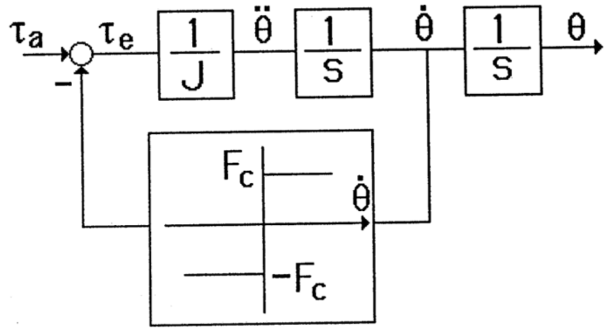


Figure B.1. Rotating member with Coulomb friction

Because of Coulomb friction the effective accelerating or decelerating torque τ_e is related to the appled torque τ_a through the equation

$$\tau_e = \tau_a \pm F_c \tag{B.1}$$

where F_c is defined as in Figure B.1. From Newton's law of motion,

$$\tau_a = F_c + J\ddot{\theta} \quad \text{for} \quad \dot{\theta} > 0 \tag{B.2}$$

$$\tau_a = -F_c + J\ddot{\theta} \quad \text{for} \quad \dot{\theta} < 0 \, . \tag{B.3}$$

$$\tau_a = M \sin\omega t \ . \tag{B.5}$$

The corresponding steady-state wave forms are sketched in Figure B.2 and B.3. The discontinuities of the τ_e wave correspond to zeros of the $\dot{\theta}$ wave because the frictional torque F_c changes sign at those instants. $\ddot{\theta}(t)$ passes through zero at

$$\omega t = n\pi - \alpha \ , \quad n = 0,1,2,\cdots, \tag{B.6}$$

while

$$\tau_a = F_c \quad \text{at} \quad \omega t = \alpha \ .$$

It follows that

$$\alpha = \sin^{-1}\lambda \tag{B.7}$$

where

$$\lambda = F_c / M \ .$$

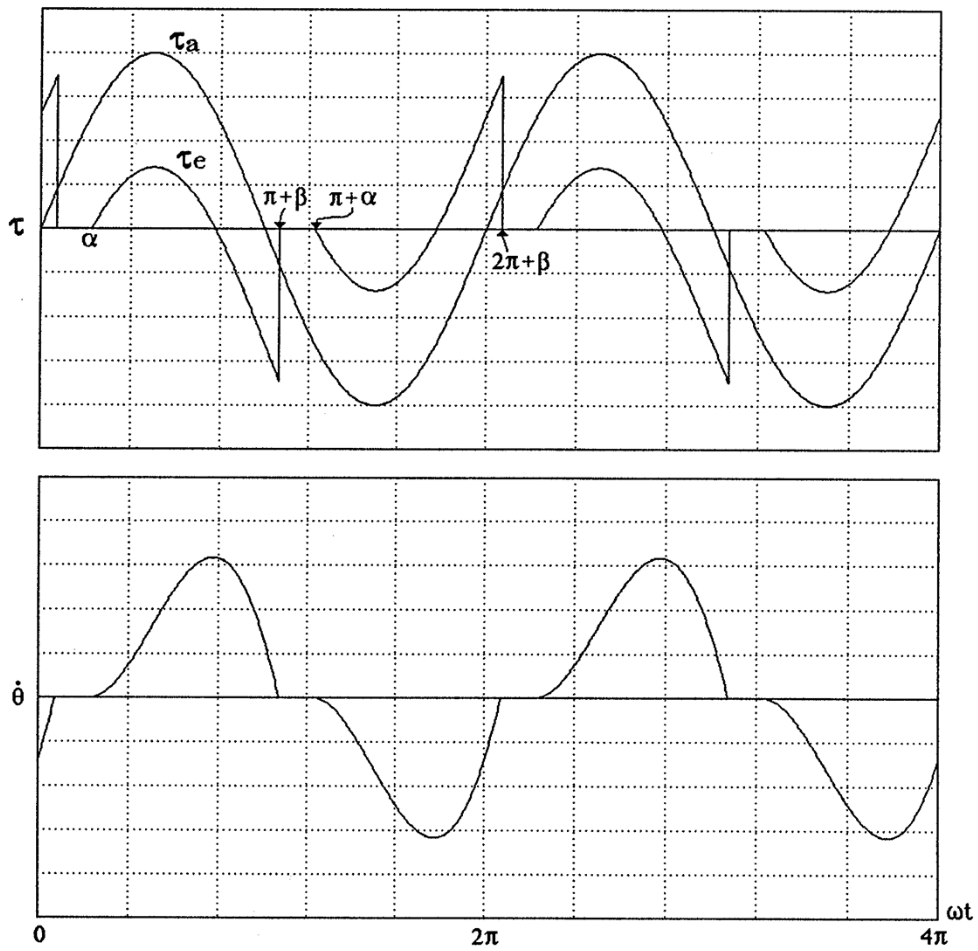


Figure B.2. Steady state wave forms with dead zones

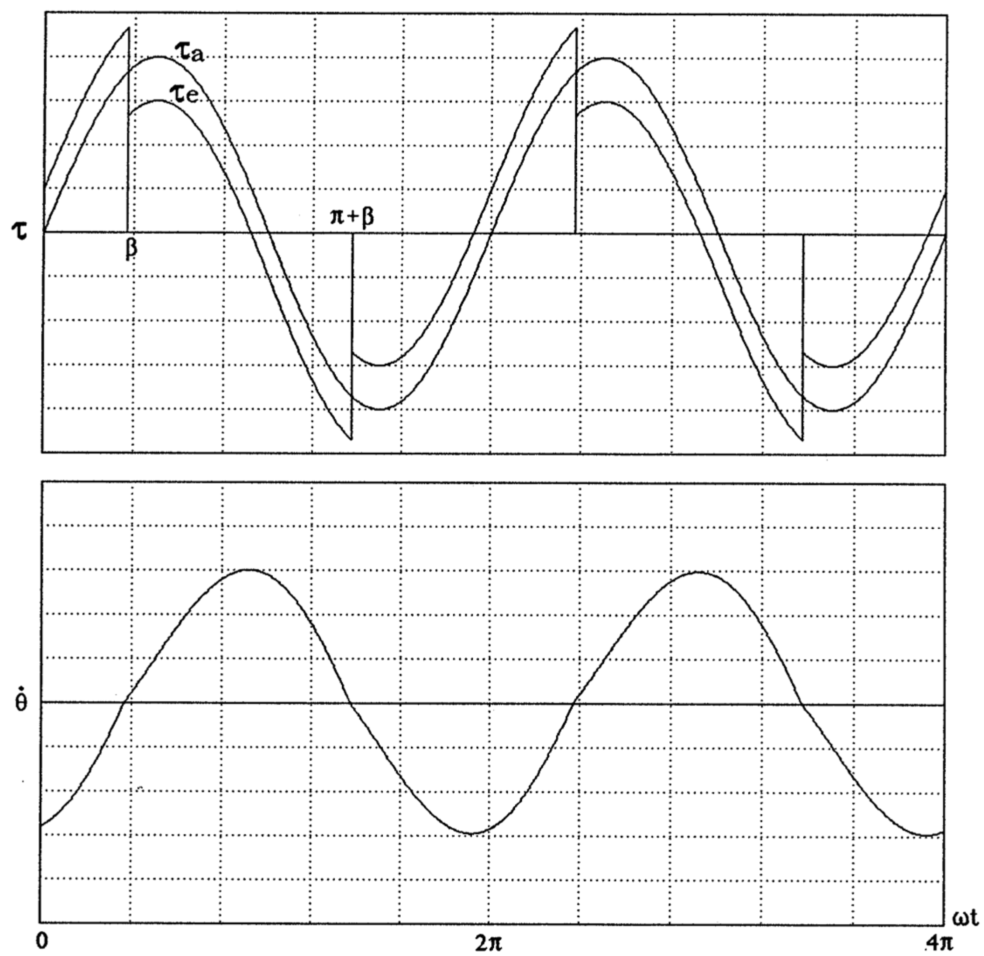


Figure B.3. Steady state wave forms without dead zones

Case 1:With dead zones.$\alpha > \beta$

As can be seen in Figure B.2, the following steady-state condition exists.

$$\int_{\alpha}^{\pi+\beta} (M \sin\omega t - F_c)\, d\,\omega t = 0 \qquad \text{(B.8)}$$

or

$$\int_{\alpha}^{\pi+\beta} (\sin\theta - \lambda)\, d\,\theta = 0$$

Simplifying eq. (B.8),

$$\beta = \frac{\cos\beta}{\lambda} + \frac{\sqrt{1-\lambda^2}}{\lambda} - \pi + \alpha \qquad \text{(B.9)}$$

Case 2:No dead zone.$\alpha \leq \beta$

From Figure B.3,

$$\int_{\beta}^{\pi+\beta} (M \sin\omega t - F_c) d\,\omega t = 0 \tag{B.10}$$

or

$$\int_{\beta} \pi+\beta(\sin\theta - \lambda) d\,\theta = 0$$

When eq. (B.10) is simplified,

$$\beta = \cos^{-1}(\pi\lambda/2) \tag{B.11}$$

Case 1:With dead zones, $\lambda > \lambda_c$ or $\alpha > \beta$

$$a_1 = \frac{1}{\pi} \int_{\alpha}^{\pi+\beta} M(\sin\omega t - \lambda)\sin(\omega t)\,d(\omega t) + \frac{1}{\pi} \int_{\pi+\alpha}^{2\pi+\beta} M(\sin\omega t + \lambda)\sin(\omega t)\,d(\omega t) \quad (B.16)$$

$$= (M/\pi)[\pi - (\alpha - \beta) - \sin\alpha(\cos\alpha + \cos\beta) - \cos\beta(\sin\alpha + \sin\beta)]$$

Similarly

$$b_1 = (M/\pi)(\sin\alpha + \sin\beta)^2 \quad (B.17)$$

Case 2:No dead zone, $\lambda \leq \lambda_c$ or $\alpha \leq \beta$

$$a_1 = \frac{2}{\pi}\int_{-\alpha}^{\beta} M(\sin\omega t + \lambda)\sin(\omega t)d(\omega t) + \frac{2}{\pi}\int_{\beta}^{\pi-\alpha} M(\sin\omega t - \lambda)\sin(\omega t)d(\omega t) \quad \text{(B.18)}$$

$$= M(1-2\lambda^2)$$

Similarly

$$b_1 = 2M\lambda\sqrt{[(2/\pi)^2 - \lambda^2]} \quad \text{(B.19)}$$

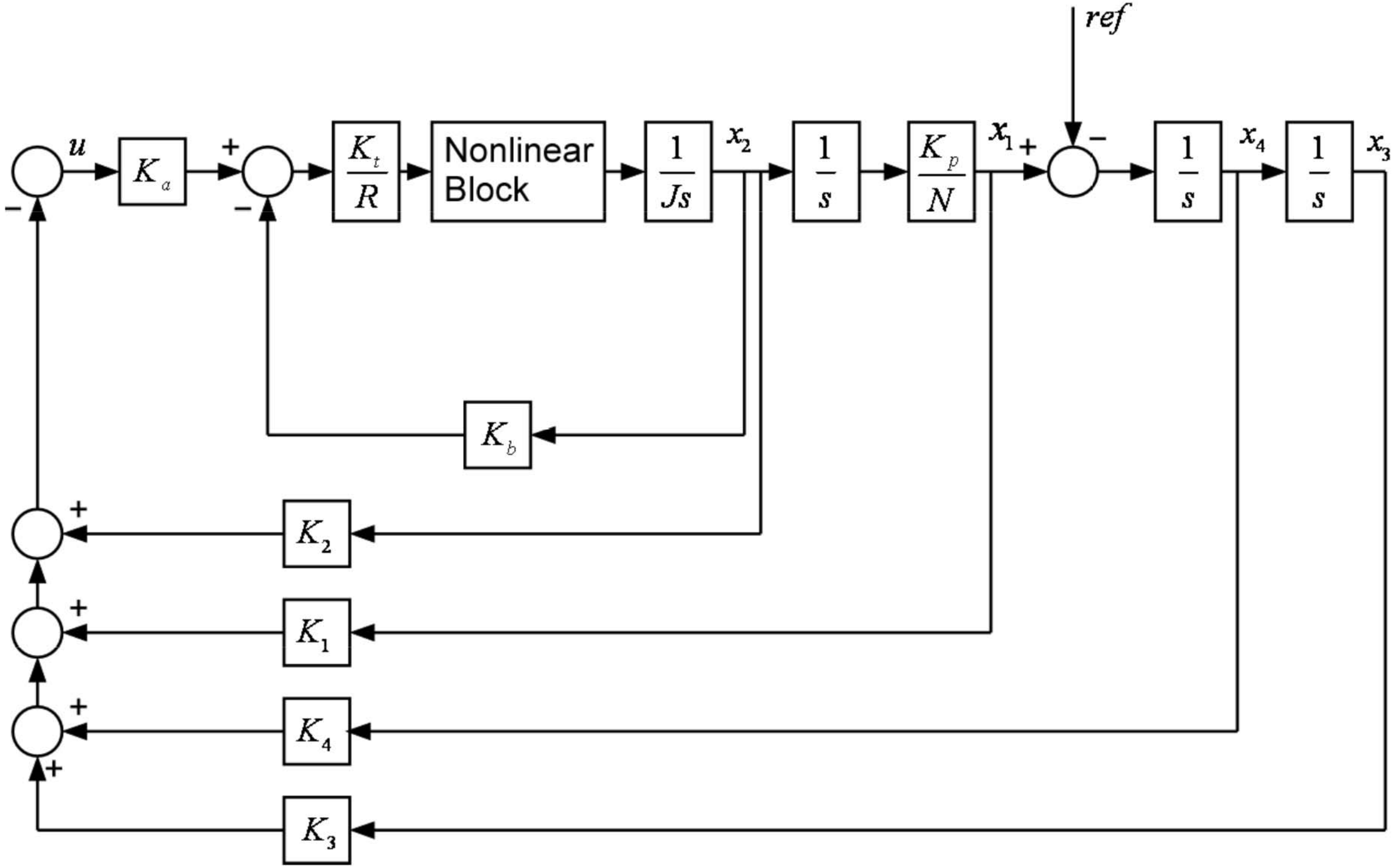
Then, the Describing Function, $N_{df}(M)$, is given by

$$N_{df}(M) = \frac{1}{M}\sqrt{a_1^2 + b_1^2}\,\exp(j\phi) \tag{B.20}$$

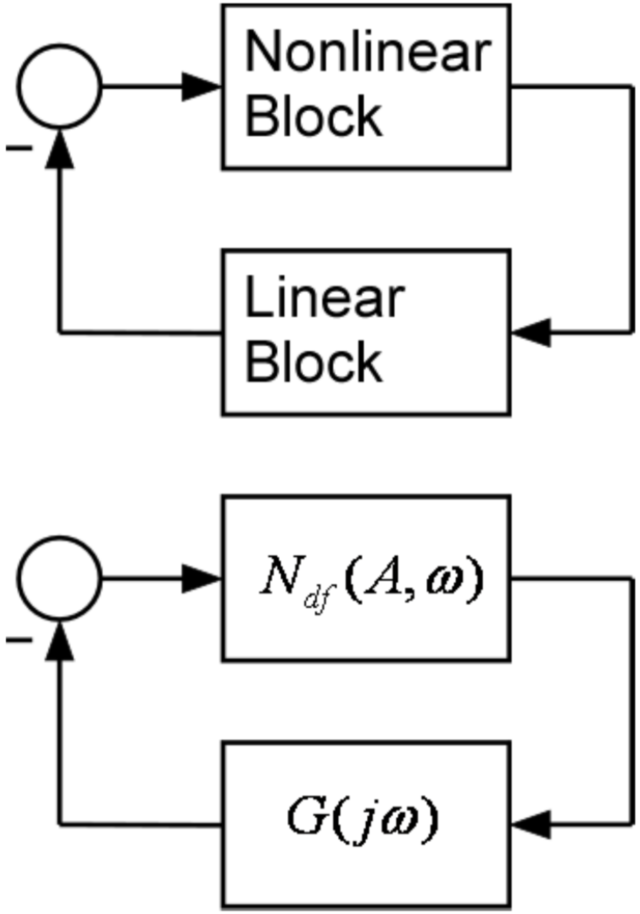
where

$$\phi = \tan^{-1}\left(\frac{b_1}{a_1}\right)$$

Tracking Controller



Limit Cycle Condition



$$1 + G(j\omega) N_{df}(A,\omega) = 0$$

$$G(j\omega) = -\frac{1}{N_{df}(A,\omega)}$$

Controller Design

```
A=[ 0 Kp/N;0 -Kt*Kb/(R*Jm) ]
B=[0;Ka*Kt/(Jm*R)]
C=[1 0]
D=[0]

A6=[A [0 0;0 0];0 0 0 1;1 0 0 0];
B6=[B;0;0];
C6=[1 0 0 0 0 0;0 0 0 0 0 1];
D6=[0;0;0];
Q6=[0 0 0 0;0 0 0 0; 0 0 1000 0;0 0 0 0];
R6=[1];
K6=lqr(A6,B6,Q6,R6)
```

Linear Block Nyquist Plot

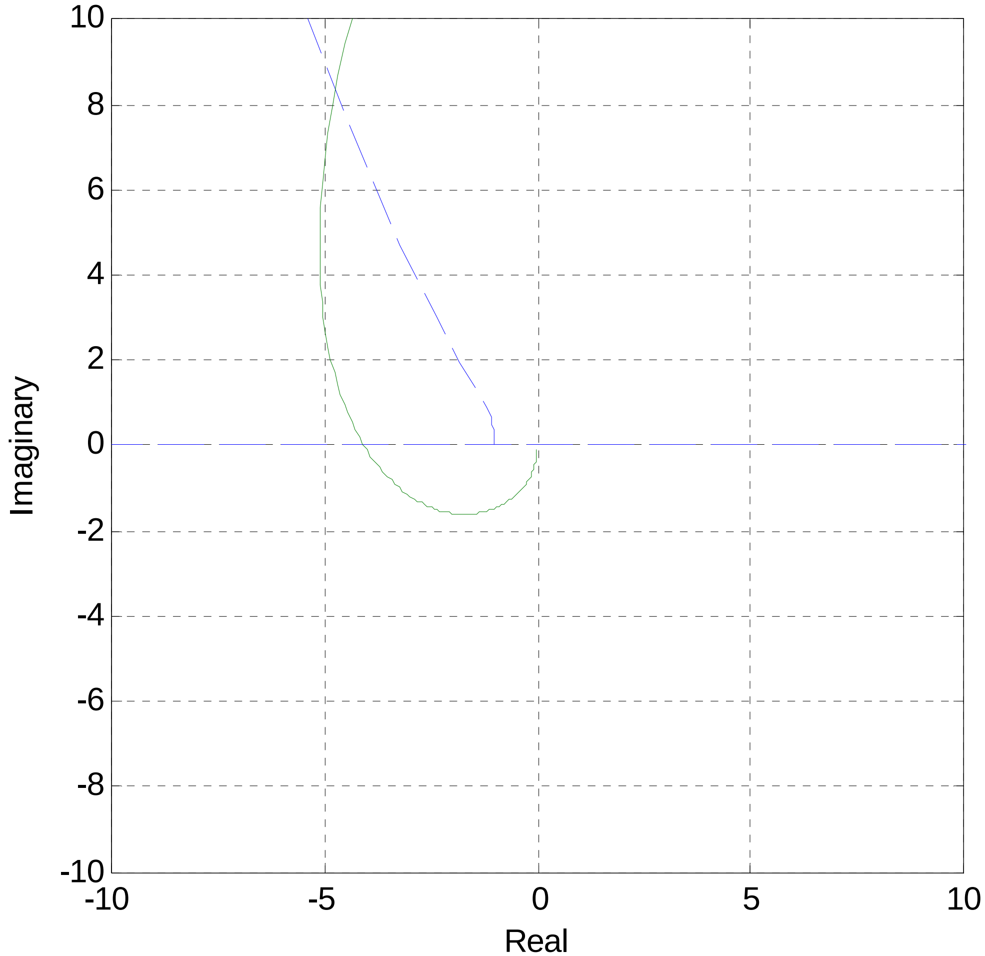
```
A1=[0 Kp/N ;0 0]
B1=[0;1/(Jm)]
C1=[0 Kt*Kb/R 0  0]

A3=[A1 [0 0;0 0];0 0 0 1;1 0 0 0];
B3=[B1;0;0]
C3=Ka*Kt*K6/R+C1

w=logspace(-1,3,500);
[re, im]=nyquist(A3,B3,C3,0,1,w);
```

Linear Block Nyquist Plot

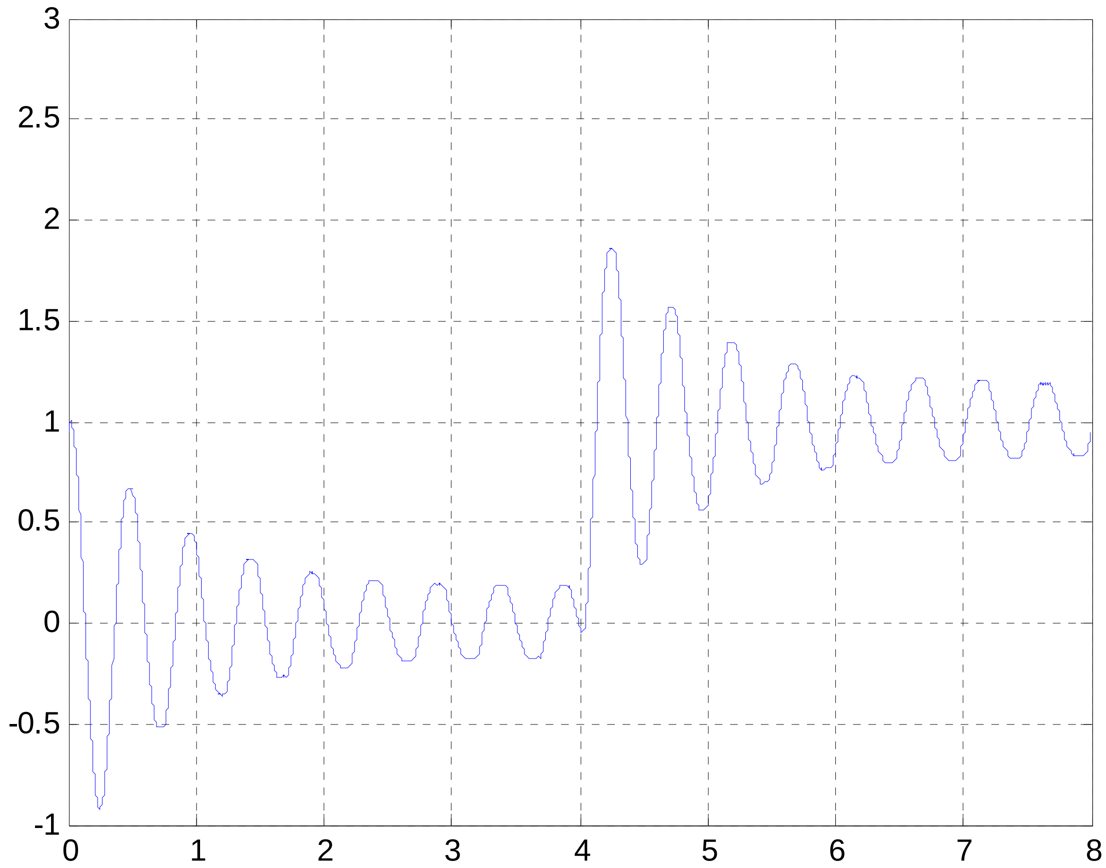
```
load  -ascii desf
plot(desf(:,1),desf(:,2),'--',re,im);
axis([-10 10 -10 10])
axis('square')
xlabel('Real');
ylabel('Imaginary');
grid
```

K6

Linear Block Nyquist Plot



Limit Cycle in Response

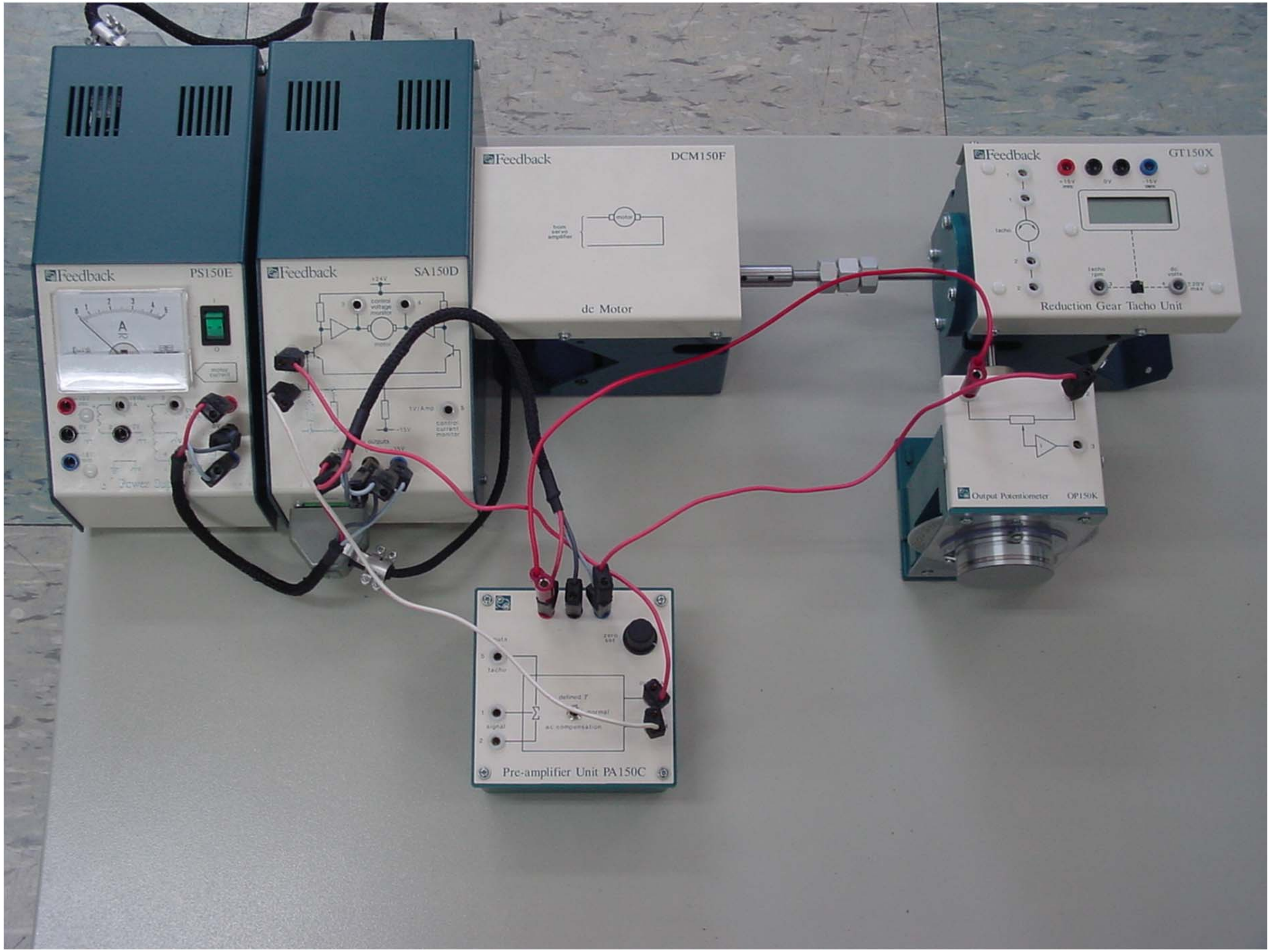


Exercise 1

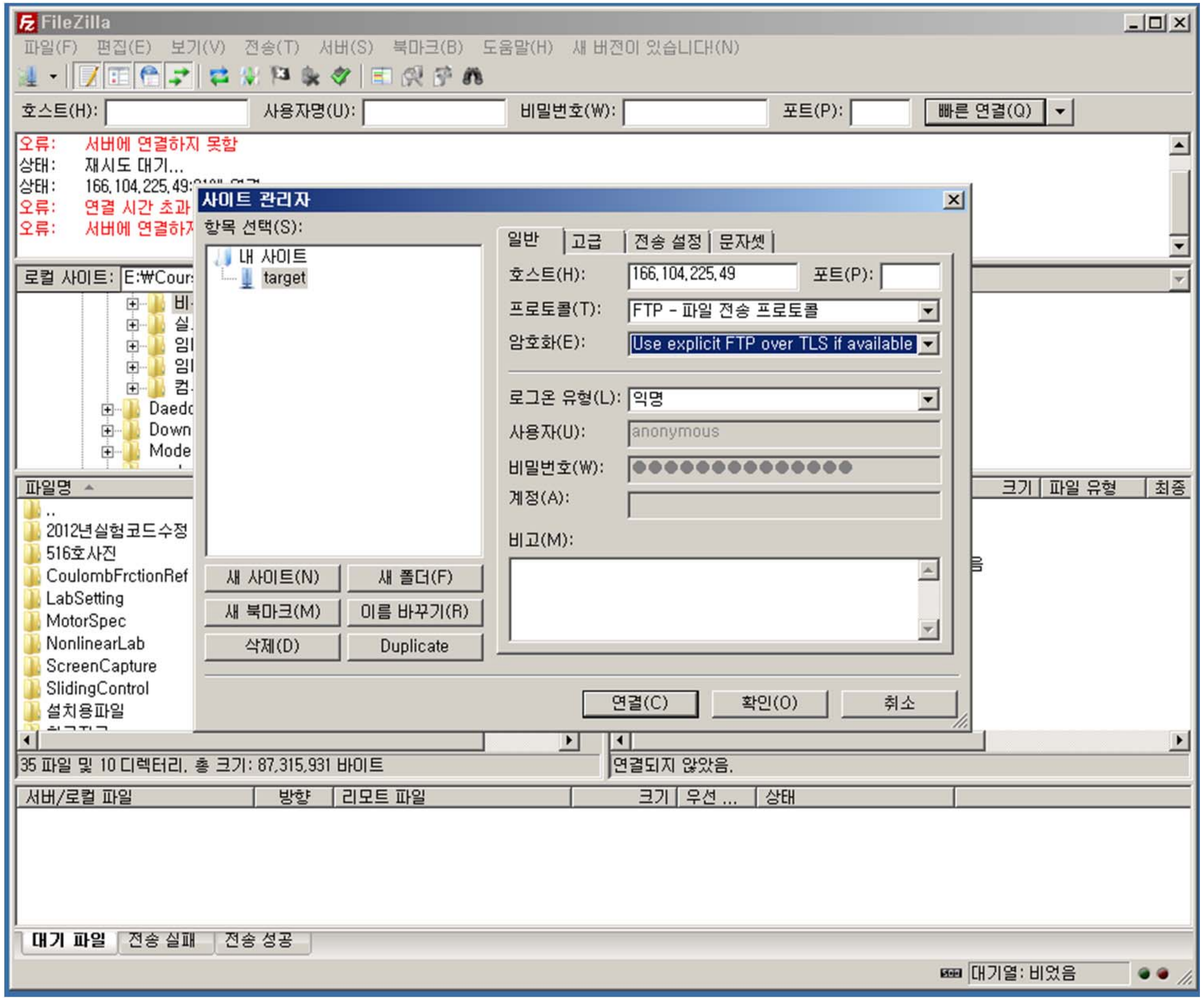
- 실험을 실행하여 리미트 사이클을 관찰하시오.
- Nyquist plot과 Describing function plot의 교차점에서 리미트 사이클의 주파수와 크기를 예측해 보고, 실험 결과와 비교해 보시오.
- MATLAB을 이용한 LQR 설계에서 Q값을 다르게 정하여 위의 실험을 반복하시오.(x1과 x2의 weight를 줄 경우 x2의 gain이 커져서 잡음의 영향이 커짐. X3의 weight는 0이 아니어야 함.)

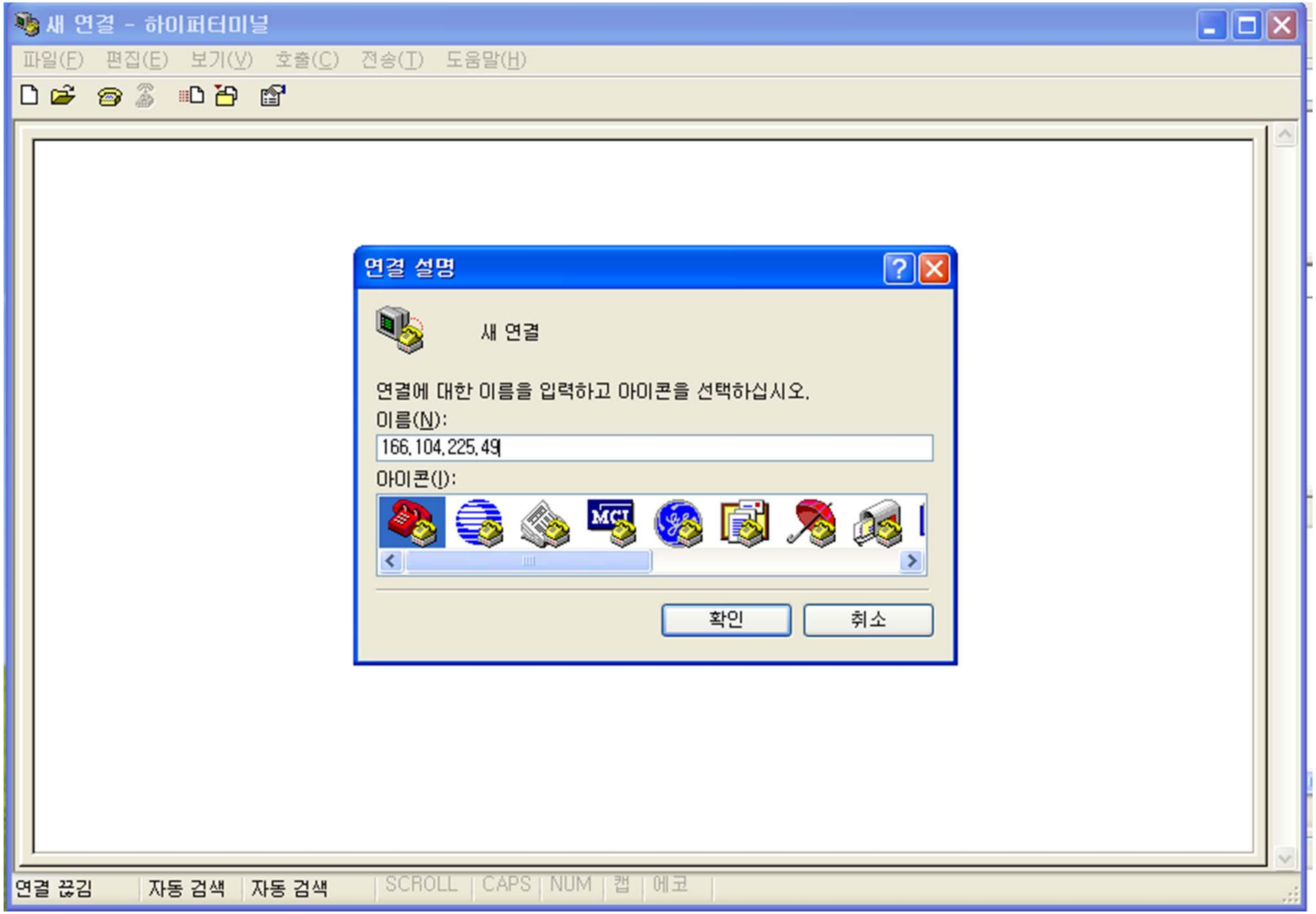
Exercise 1

- Observe the limit cycle.
- Predict the frequency and amplitude of the limit cycle from the intersection of Nyquist plot and describing function curve. Compare the predicted results with the experiments.
- Repeat the above experiment with the different values of Q.(If you increase the weights of x1 and x2, the influence of noise will be increased due to the large gain of x2. The weight of x3 must be nonzero.)

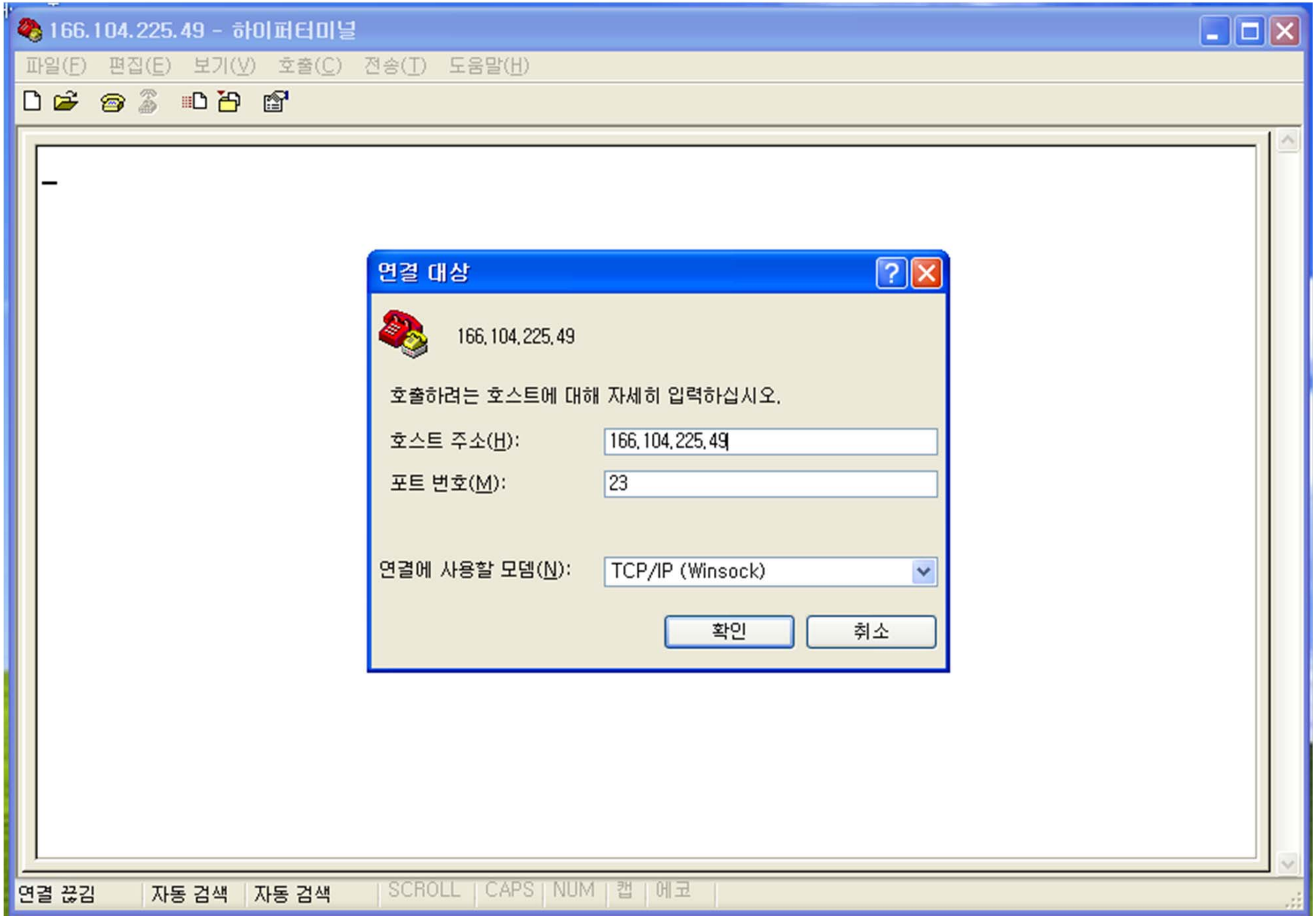

Feedback
PS150E
Feedback
SA150D
Feedback
DCM150F
dc Motor
Feedback
GT150X
Reduction Gear Tacho Unit
Output Potentiometer
OP150K
Pre-amplifier Unit PA150C

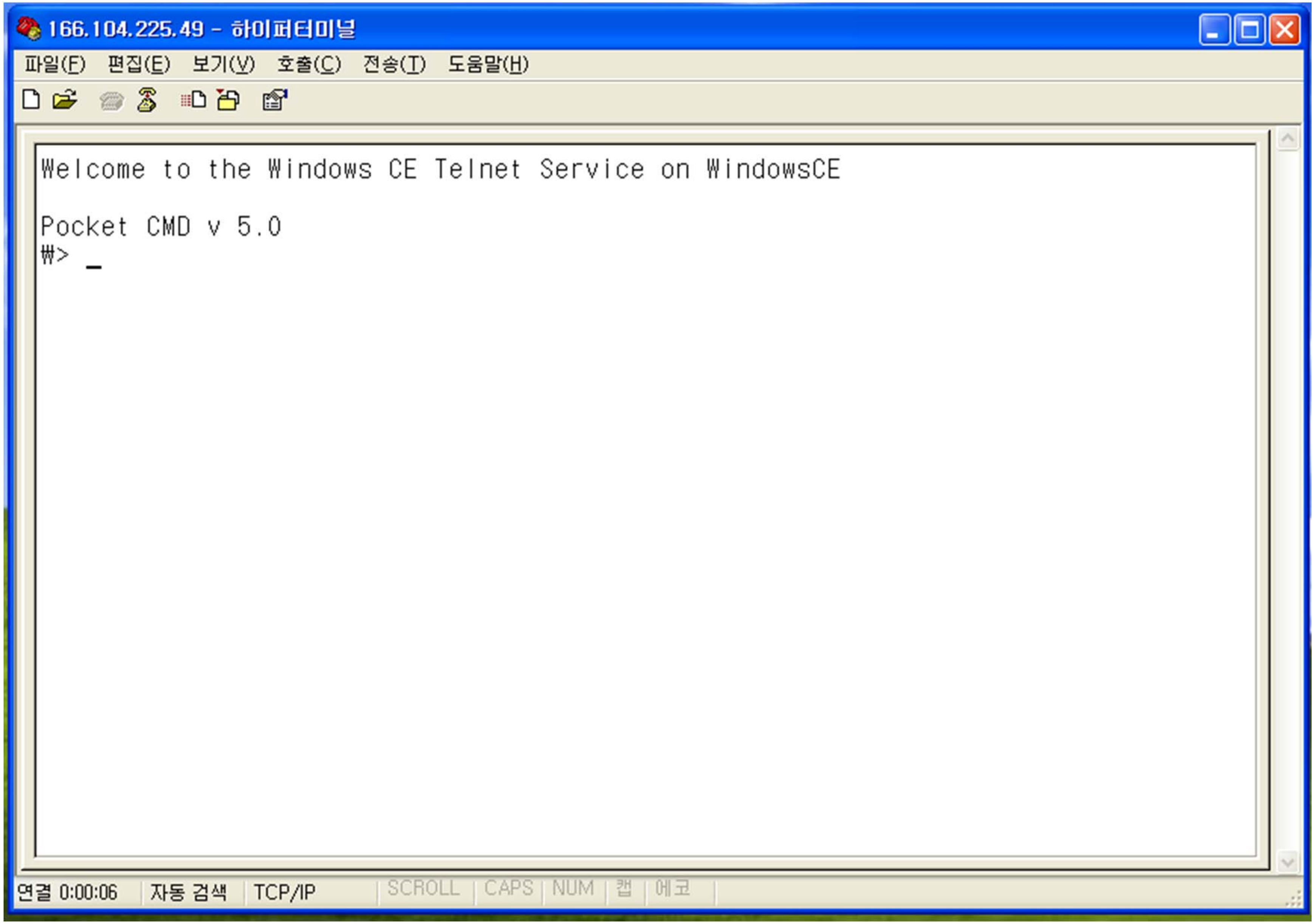
Connect FTP before experiment



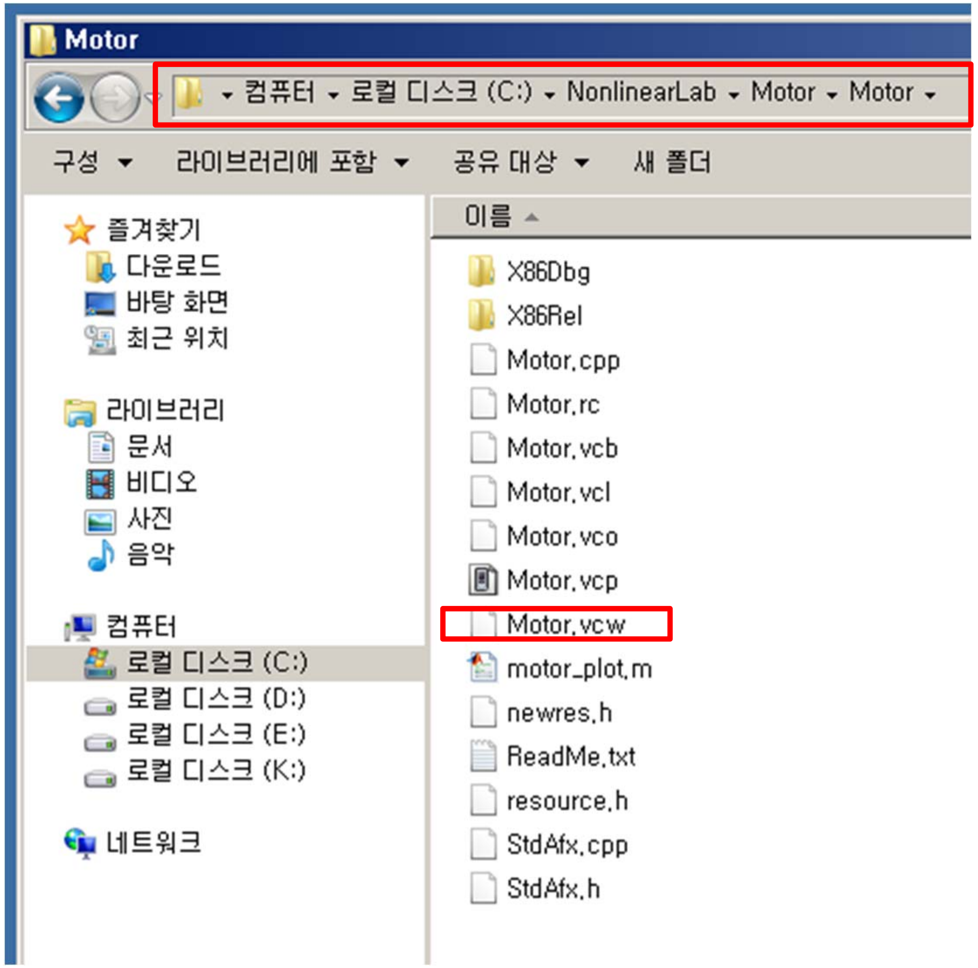


새 연결 - 하이퍼터미널
파일(F) 편집(E) 보기(V) 호출(C) 전송(T) 도움말(H)
연결 설명
새 연결
연결에 대한 이름을 입력하고 아이콘을 선택하십시오.
이름(N):
166.104.225.49
아이콘(I):
MCI
확인
취소
연결 끊김
자동 검색
자동 검색
SCROLL
CAPS
NUM
캡
에코

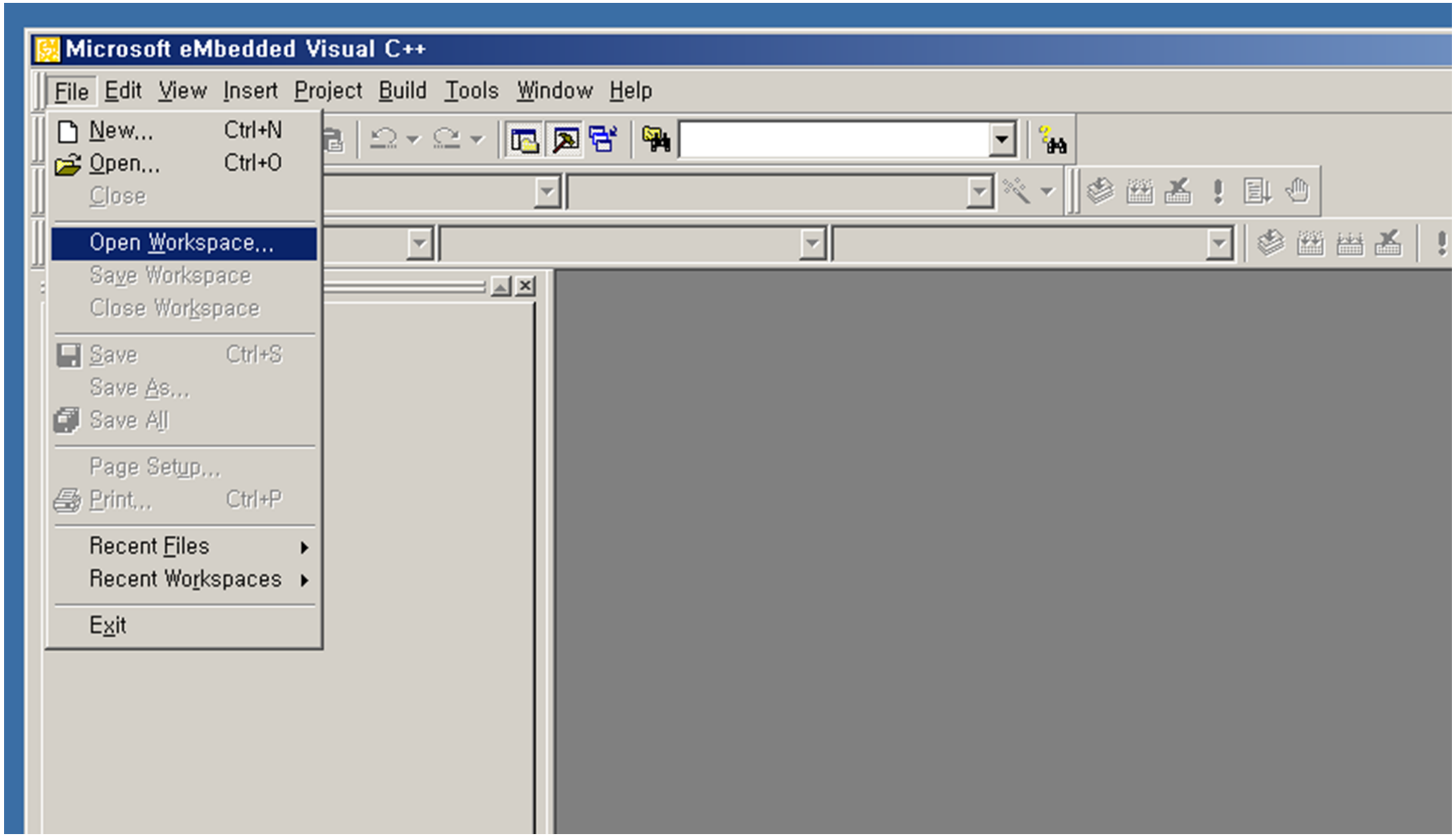

166.104.225.49 - 하이퍼터미널
파일(F) 편집(E) 보기(V) 호출(C) 전송(T) 도움말(H)
연결 대상
166.104.225.49
호출하려는 호스트에 대해 자세히 입력하십시오.
호스트 주소(H): 166.104.225.49
포트 번호(M): 23
연결에 사용할 모뎀(N): TCP/IP (Winsock)
확인
취소
연결 끊김
자동 검색
자동 검색
SCROLL
CAPS
NUM
캡
에코


166.104.225.49 - 하이퍼터미널
파일(F) 편집(E) 보기(V) 호출(C) 전송(T) 도움말(H)
Welcome to the Windows CE Telnet Service on WindowsCE
Pocket CMD v 5.0
₩> _
연결 0:00:06 자동 검색 TCP/IP SCROLL CAPS NUM 캡 에코

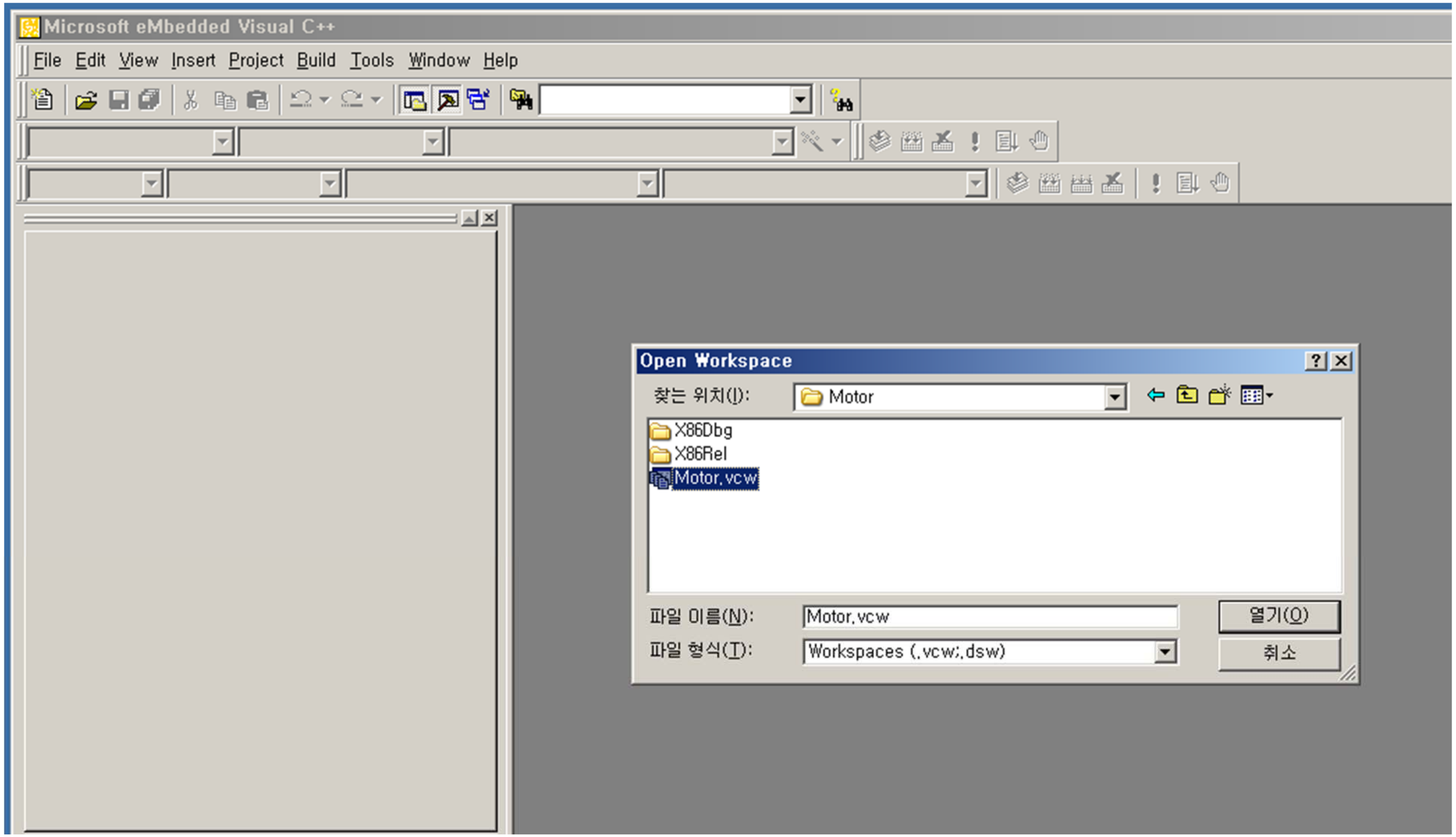
Find project workspace file



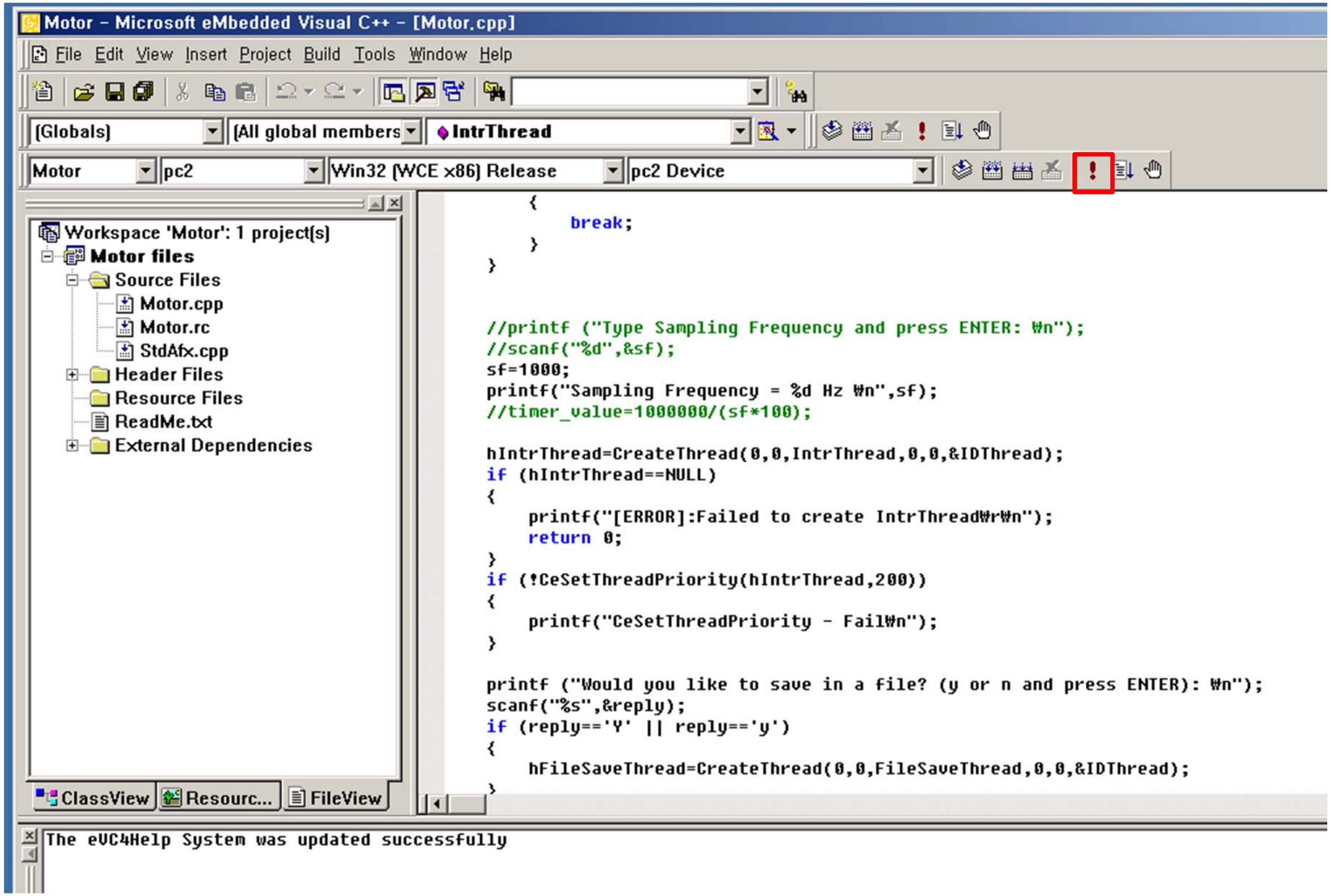
Open Workspace from File menu



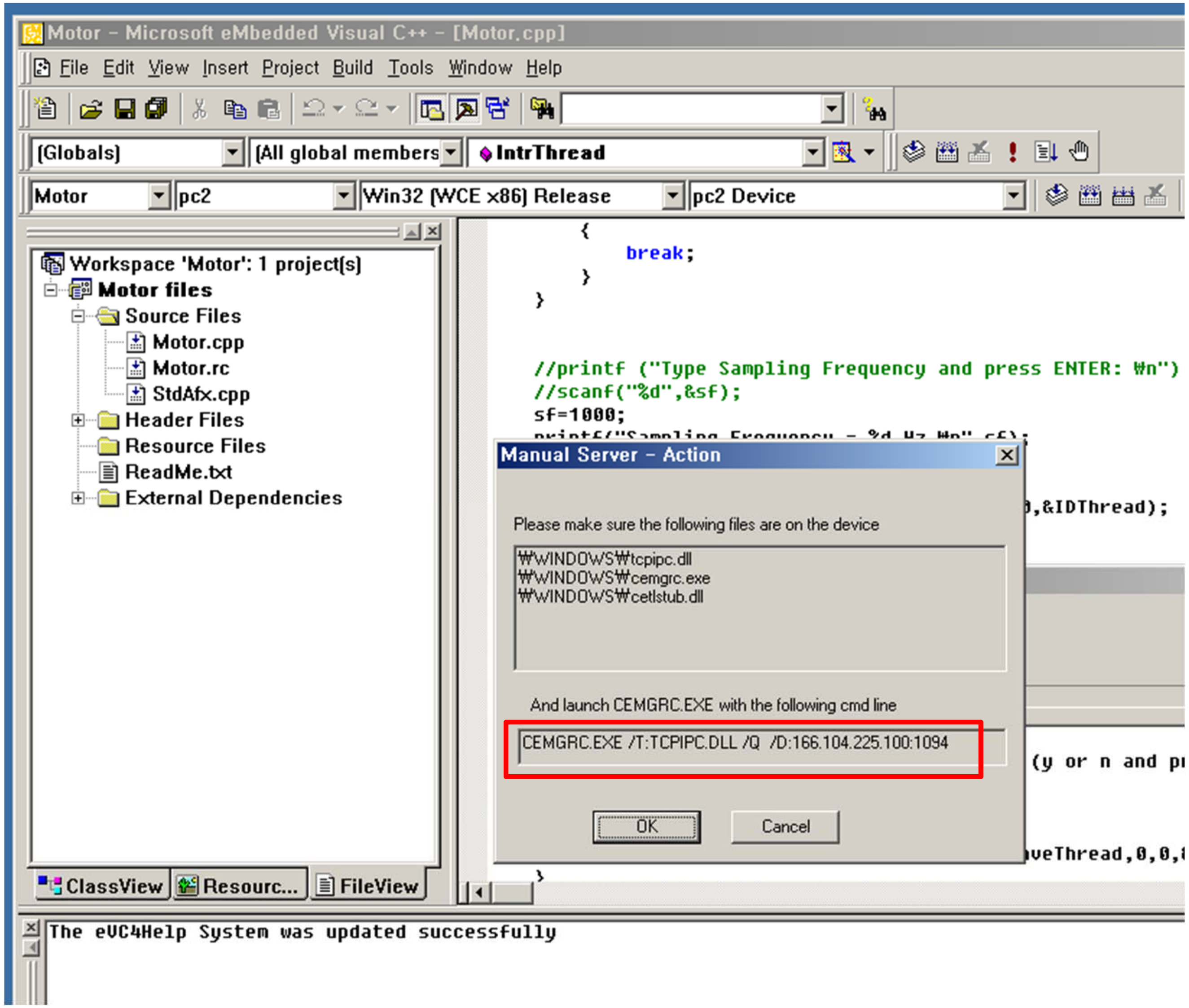
Open Workspace



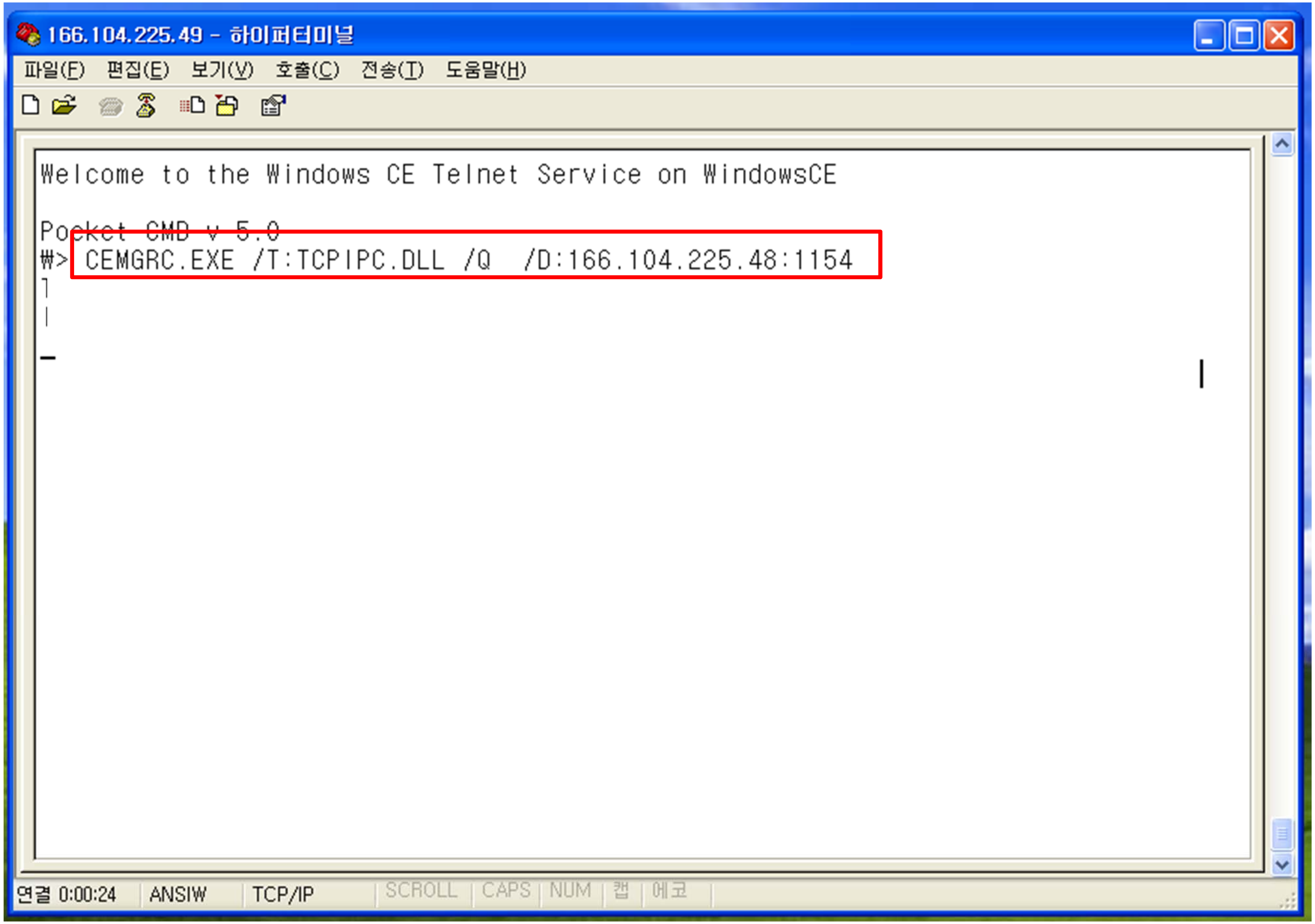
Press the execute button



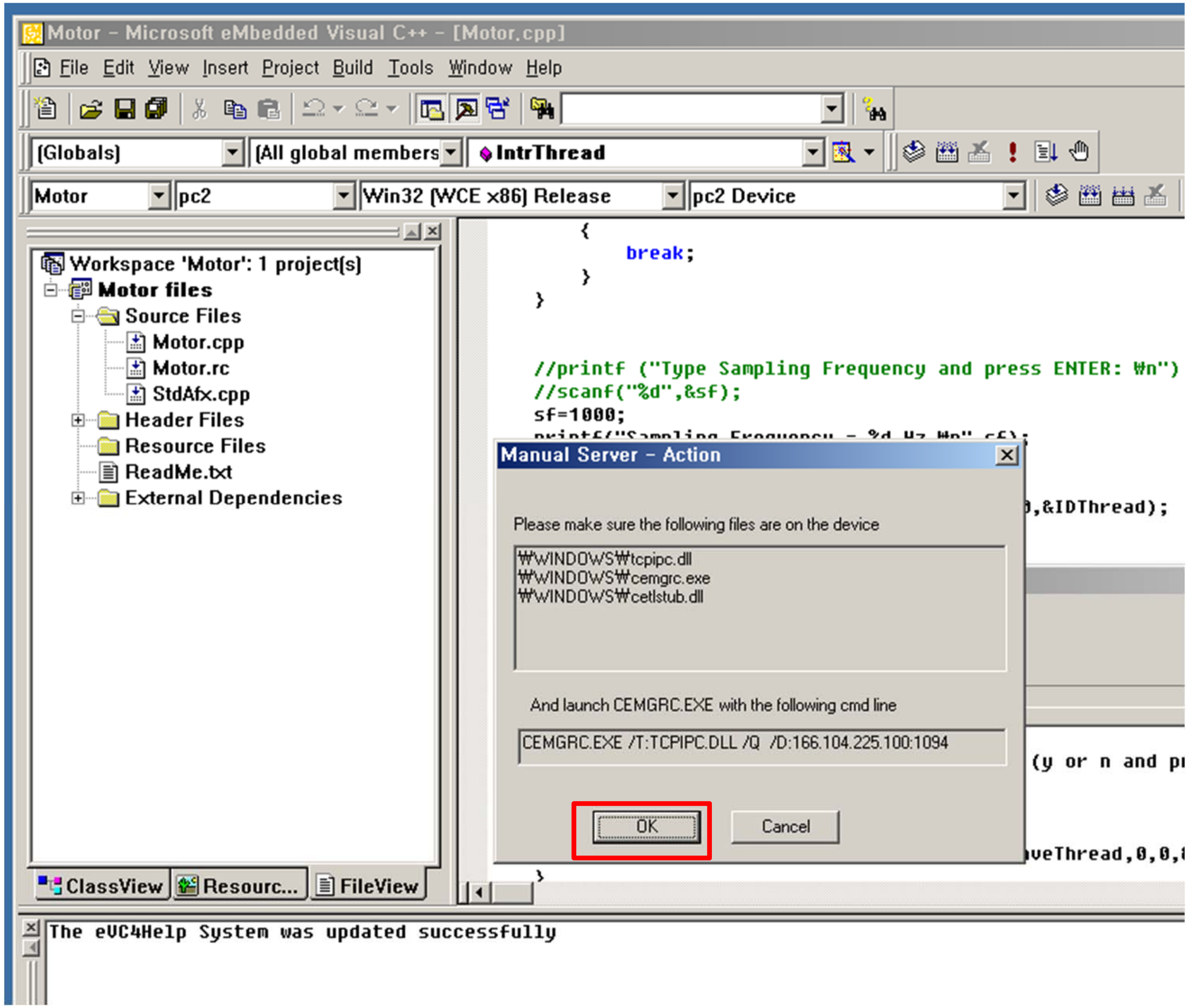
Copy the command

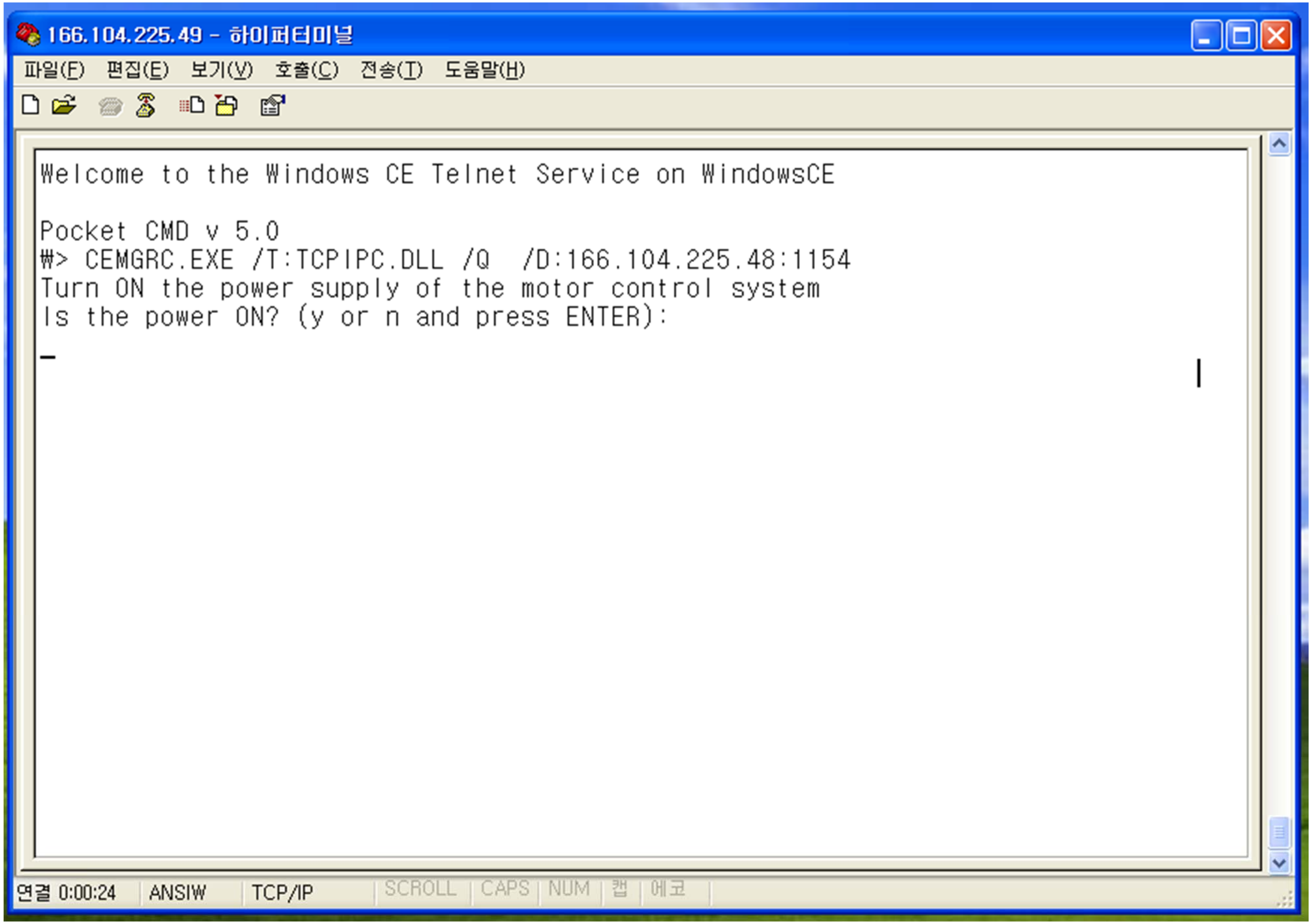


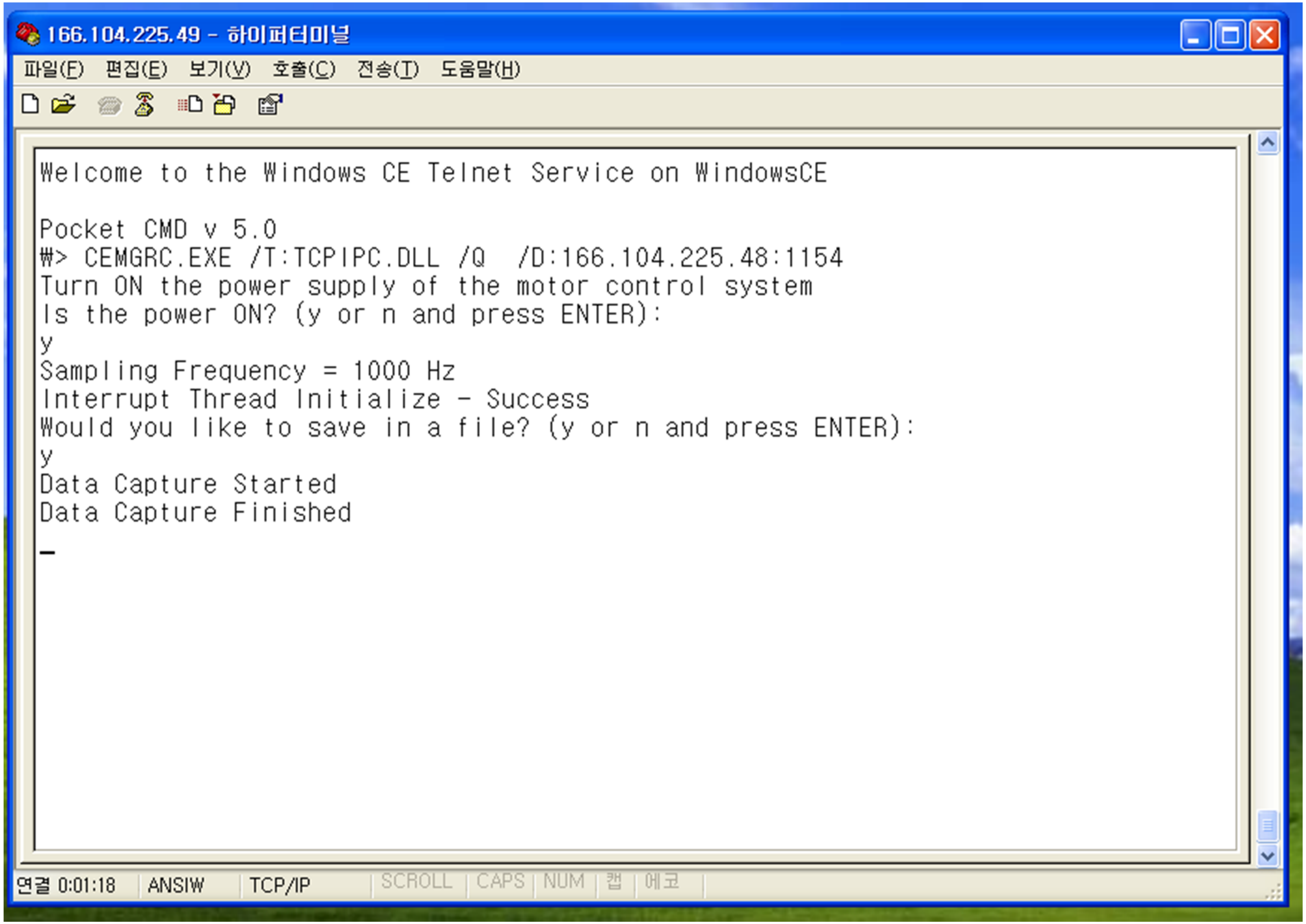
Paste and Enter on the Hyperterminal



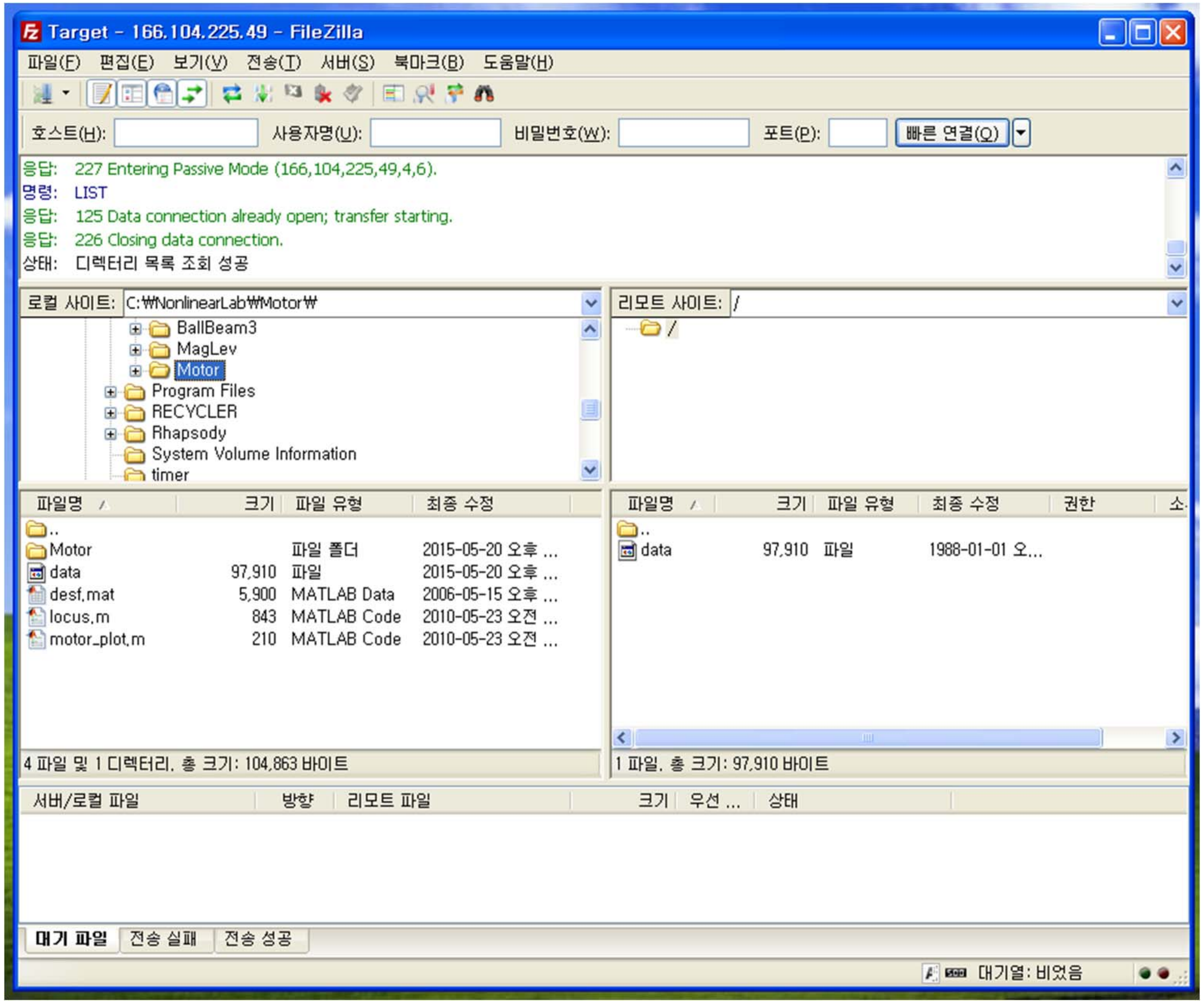
Then, press OK



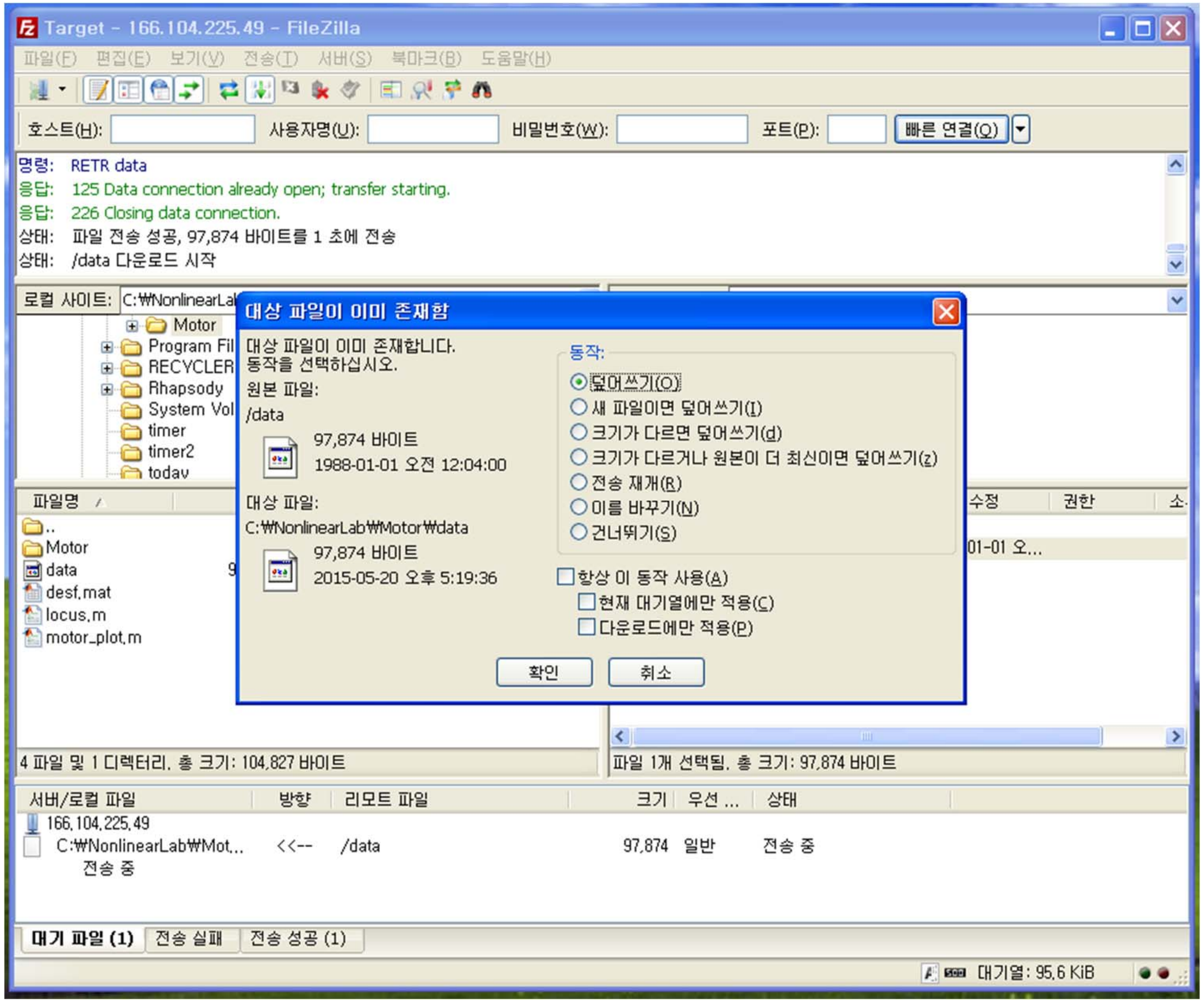

166.104.225.49 - 하이퍼터미널
파일(F) 편집(E) 보기(V) 호출(C) 전송(T) 도움말(H)
Welcome to the Windows CE Telnet Service on WindowsCE
Pocket CMD v 5.0
₩> CEMGRC.EXE /T:TCPIPC.DLL /Q /D:166.104.225.48:1154
Turn ON the power supply of the motor control system
Is the power ON? (y or n and press ENTER):
연결 0:00:24
ANSIW
TCP/IP
SCROLL
CAPS
NUM
캡
에코

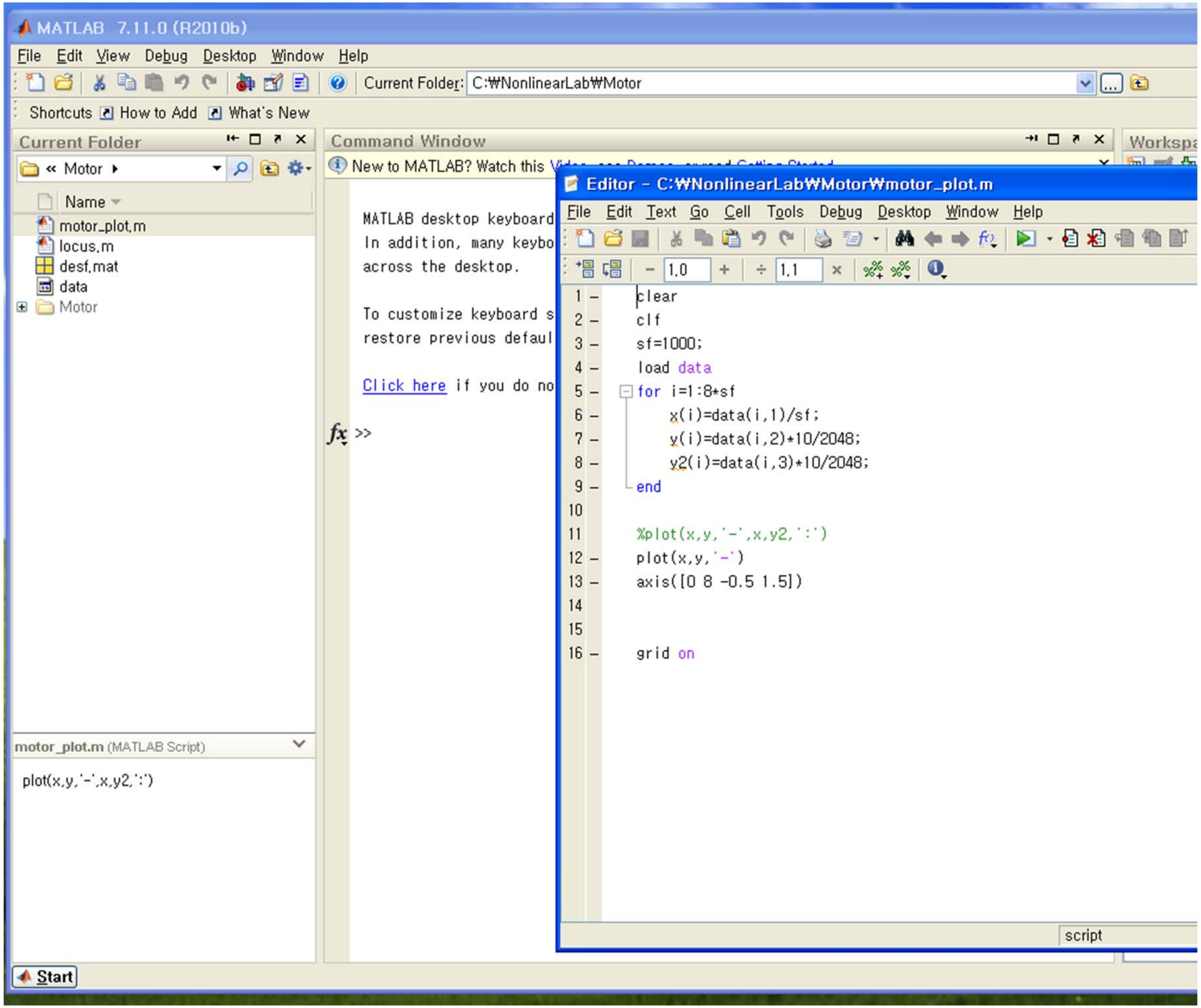

166.104.225.49 - 하이퍼터미널
파일(F) 편집(E) 보기(V) 호출(C) 전송(T) 도움말(H)
Welcome to the Windows CE Telnet Service on WindowsCE
Pocket CMD v 5.0
₩> CEMGRC.EXE /T:TCPIPC.DLL /Q /D:166.104.225.48:1154
Turn ON the power supply of the motor control system
Is the power ON? (y or n and press ENTER):
y
Sampling Frequency = 1000 Hz
Interrupt Thread Initialize - Success
Would you like to save in a file? (y or n and press ENTER):
y
Data Capture Started
Data Capture Finished
연결 0:01:18 ANSIW TCP/IP SCROLL CAPS NUM 캡 에코

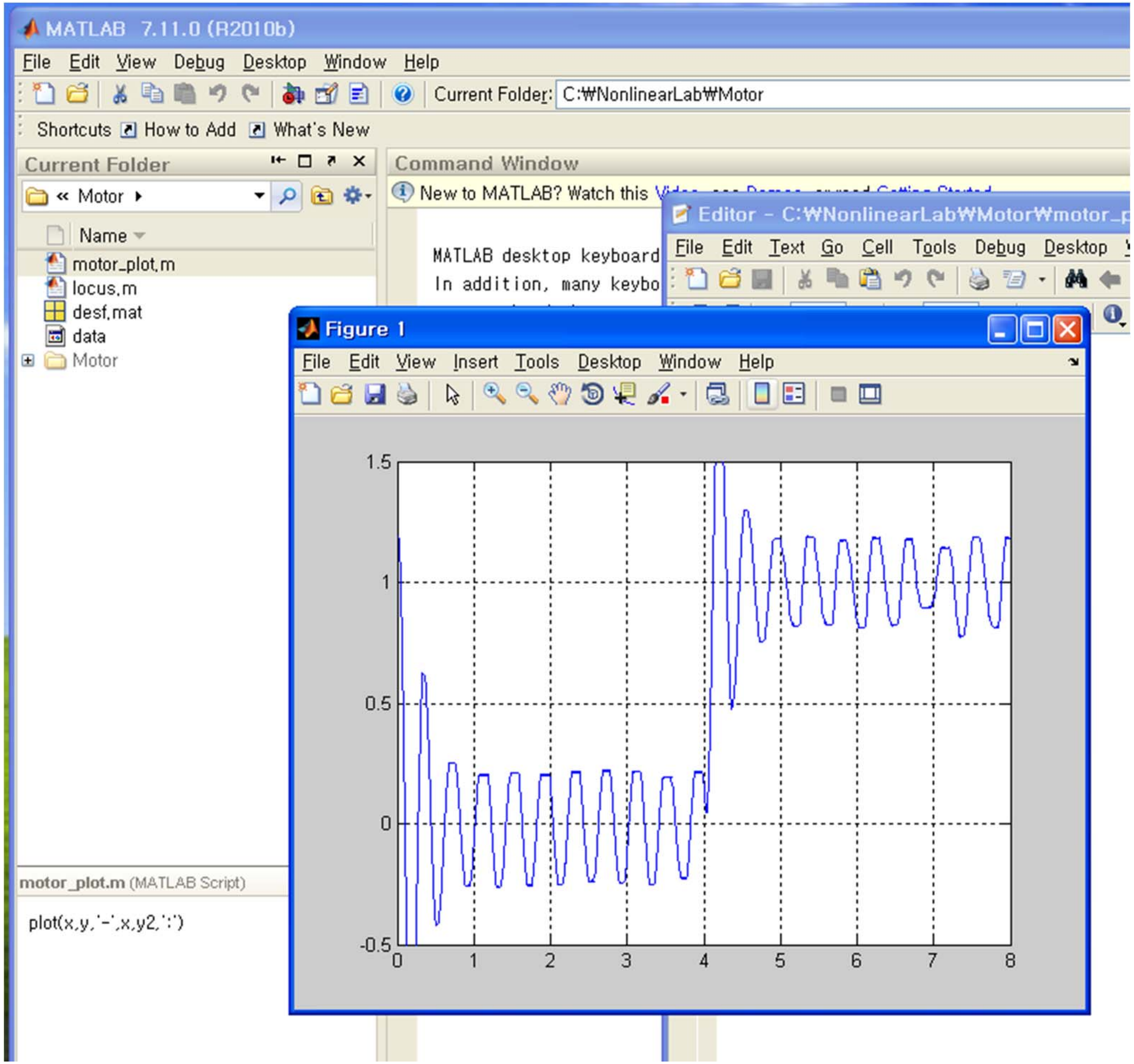
Double click data file on remote



Overwrite the file

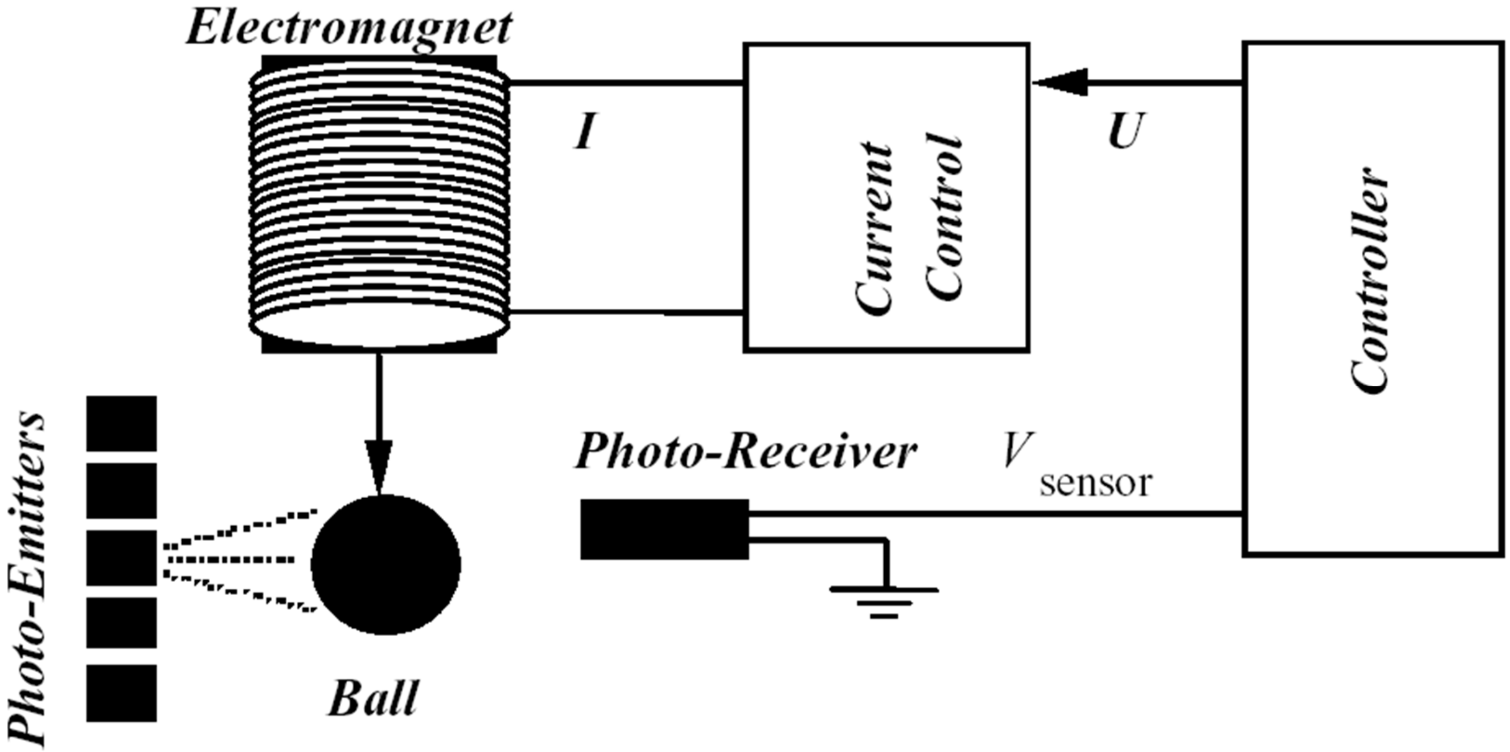



MATLAB 7.11.0 (R2010b)
File Edit View Debug Desktop Window Help
Current Folder: C:₩NonlinearLab₩Motor
Shortcuts How to Add What's New
Current Folder
Motor
Name
motor_plot.m
locus.m
desf.mat
data
Motor
motor_plot.m (MATLAB Script)
plot(x,y,'-',x,y2,':')
Command Window
New to MATLAB? Watch this
MATLAB desktop keyboard
In addition, many keybo
across the desktop.
To customize keyboard s
restore previous defaul
Click here if you do no
Editor - C:₩NonlinearLab₩Motor₩motor_plot.m
File Edit Text Go Cell Tools Debug Desktop Window Help
clear
clf
sf=1000;
load data
for i=1:8*sf
x(i)=data(i,1)/sf;
y(i)=data(i,2)*10/2048;
y2(i)=data(i,3)*10/2048;
end
%plot(x,y,'-',x,y2,':')
plot(x,y,'-')
axis([0 8 -0.5 1.5])
grid on
script
Start


MATLAB 7.11.0 (R2010b)
File Edit View Debug Desktop Window Help
Current Folder: C:₩NonlinearLab₩Motor
Shortcuts How to Add What's New
Current Folder
Motor
Name
motor_plot.m
locus.m
desf.mat
data
Motor
motor_plot.m (MATLAB Script)
plot(x,y,'-',x,y2,':')
Command Window
New to MATLAB? Watch this
MATLAB desktop keyboard
In addition, many keybo
Editor - C:₩NonlinearLab₩Motor₩motor_p
File Edit Text Go Cell Tools Debug Desktop
Figure 1
File Edit View Insert Tools Desktop Window Help
1.5
1
0.5
0
-0.5
0 1 2 3 4 5 6 7 8

Magnetic Levitation

Magnetic Levitation System



Dynamic Equation

$$m\ddot{x} = mg - k\frac{i^2}{x^2}$$

$$i = 0.15u + I_0$$

$$v = \gamma(x - X_0)$$

m: ball mass

g: gravitational acceleration

γ: sensor gain

x: ball position from the magnet

i: input current for magnet

v: sensor output

u: voltage input for the current amplifier

$$k = \frac{mgX_0^2}{I_0^2}$$

State Equation

$$x_1 = v$$

$$x_2 = \dot{v} = \gamma \dot{x}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} f_1(x_1, x_2, u) \\ f_2(x_1, x_2, u) \end{bmatrix} = \begin{bmatrix} x_2 \\ \gamma g - \dfrac{\gamma k}{m} \dfrac{(0.15u + I_0)^2}{(x_1 / \gamma + X_0)^2} \end{bmatrix}$$

Lyapunov Linearization

$$x_1 = 0, x_2 = 0, u = 0$$

$$\begin{bmatrix} \Delta\dot{x}_1 \\ \Delta\dot{x}_2 \end{bmatrix} = A \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} + B\Delta u$$

$$A = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \dfrac{\partial f_1}{\partial x_2} \\ \dfrac{\partial f_2}{\partial x_1} & \dfrac{\partial f_2}{\partial x_2} \end{bmatrix}\Bigg|_{x_1 = 0, x_2 = 0, u = 0}$$

$$= \begin{bmatrix} 0 & 1 \\ \dfrac{2k}{m}(0.15u + I_0^2)^2(\dfrac{x_1}{\gamma} + X_0)^{-3} & 0 \end{bmatrix}\Bigg|_{x_1 = 0, x_2 = 0, u = 0} = \begin{bmatrix} 0 & 1 \\ \dfrac{2g}{X_0} & 0 \end{bmatrix}$$

Lyapunov Linearization

$$B = \begin{bmatrix} \dfrac{\partial f_1}{\partial u} \\ \dfrac{\partial f_2}{\partial u} \end{bmatrix} \Bigg|_{x_1 = 0, x_2 = 0, u = 0}$$

$$= \begin{bmatrix} 0 \\ -\dfrac{0.3\gamma k}{m} \dfrac{(0.15u + I_0)}{(x_1/\gamma + X_0)^2} \end{bmatrix} \Bigg|_{x_1 = 0, x_2 = 0, u = 0} = \begin{bmatrix} 0 \\ -\dfrac{0.3\gamma g}{I_0} \end{bmatrix}$$

$$G(s) = -\frac{\eta}{s^2 - \omega_0^2} \qquad \eta = \frac{0.3\gamma g}{I_0}$$

$$= -\frac{1200}{s^2 - 852} \qquad \omega_0 = \sqrt{\frac{2g}{X_0}}$$

Parameters

$$\gamma = 1000/3 \;\; V/m$$

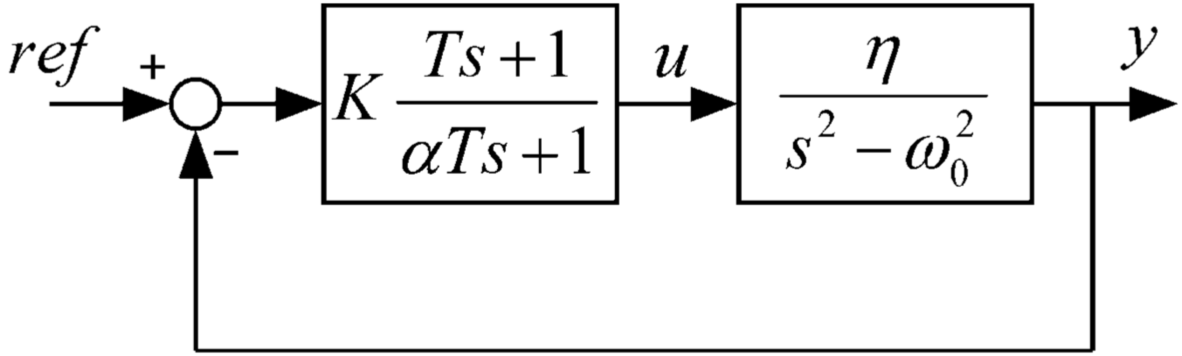
$$g = 9.8 \;\; m/\sec^2$$

$$I_0 = 0.817 \;\; A$$

$$X_0 = 23/1000 \;\; m$$

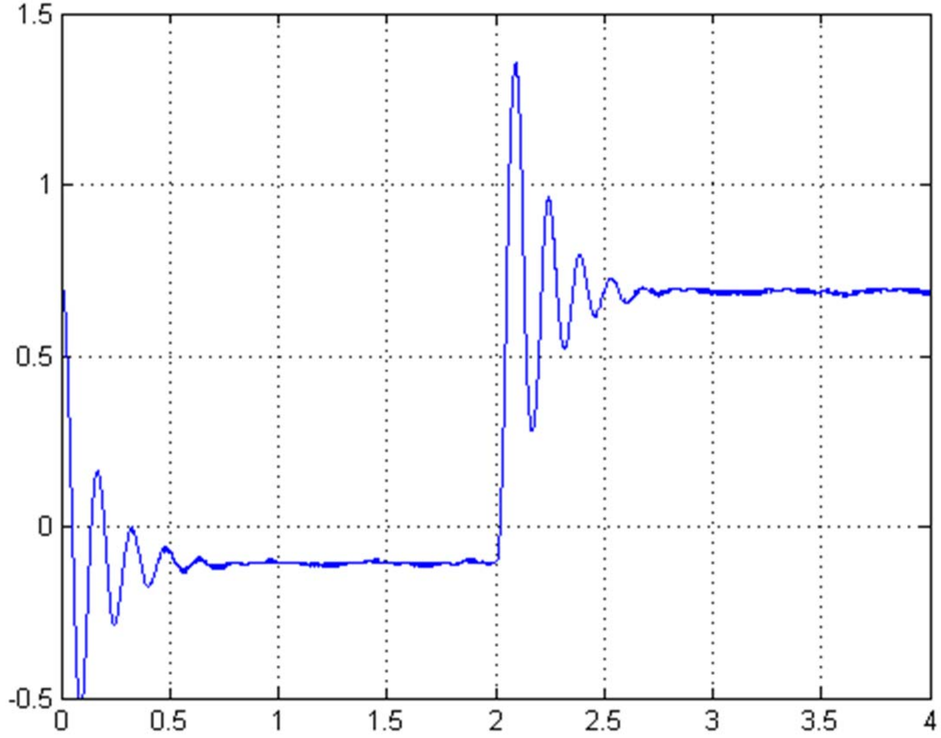
$$m = 21/1000 \; Kg$$

Lead Compensator



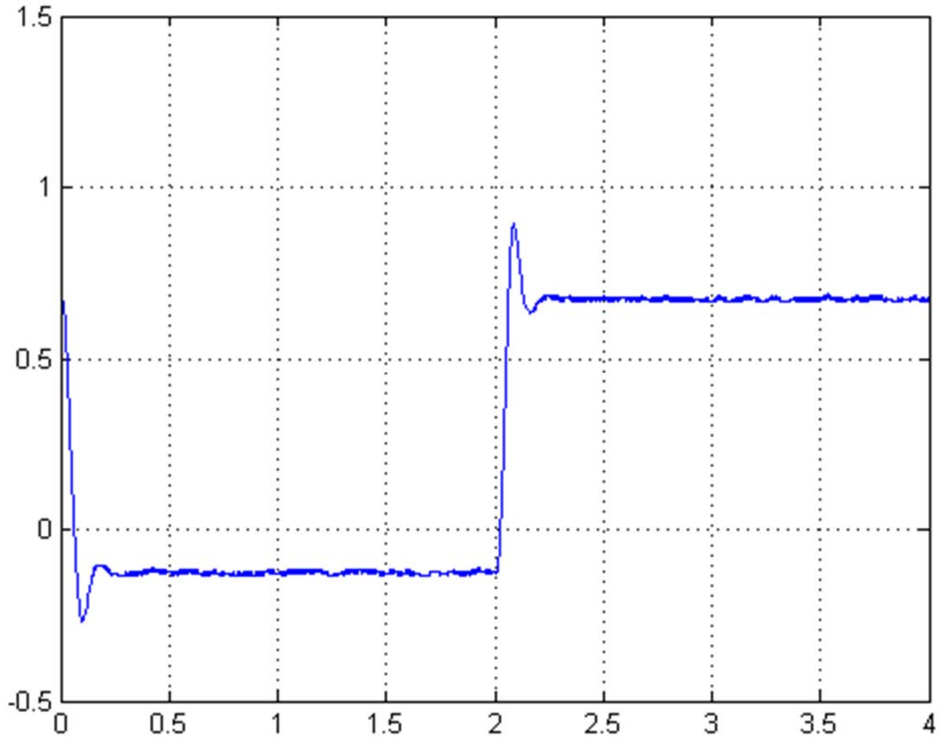
Lead Compensator

$$D_1(s) = 2\frac{0.01s+1}{0.001s+1}$$



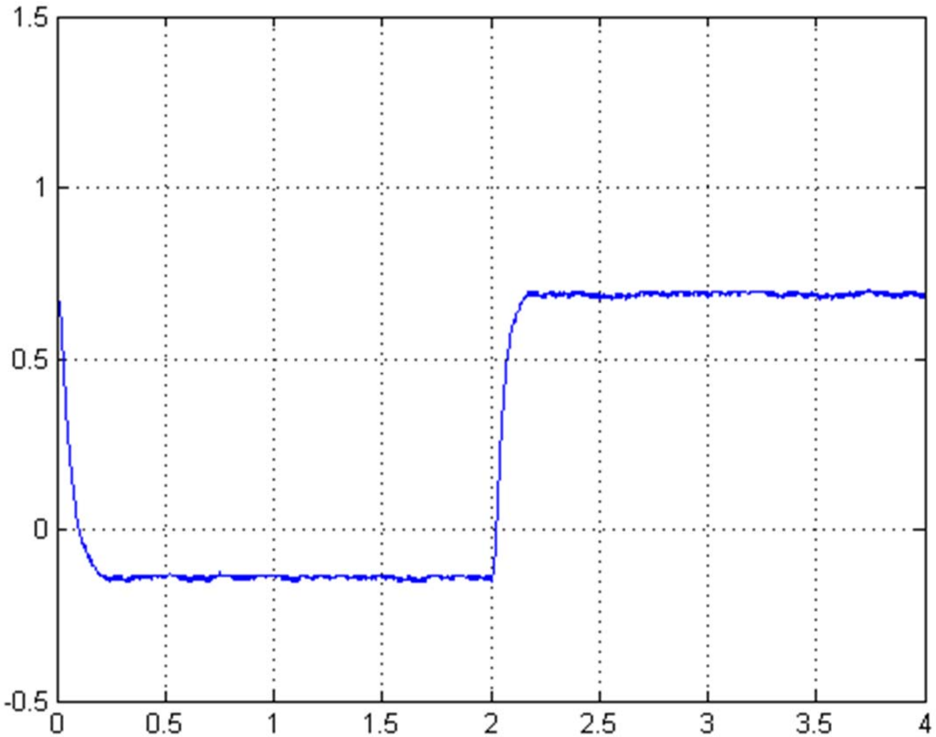
Lead Compensator

$$D_2(s) = 2\frac{0.02s+1}{0.001s+1}$$



Lead Compensator

$$D_3(s) = 2\frac{0.04s+1}{0.001s+1}$$



Input-Output Linearization

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} f_1(x_1, x_2, u) \\ f_2(x_1, x_2, u) \end{bmatrix} = \begin{bmatrix} x_2 \\ \gamma g - \dfrac{\gamma k}{m} \dfrac{(0.15u + I_0)^2}{(x_1/\gamma + X_0)^2} \end{bmatrix}$$

$$w = \gamma g - \frac{\gamma k}{m} \frac{(0.15u + I_0)^2}{(x_1/\gamma + X_0)^2}$$

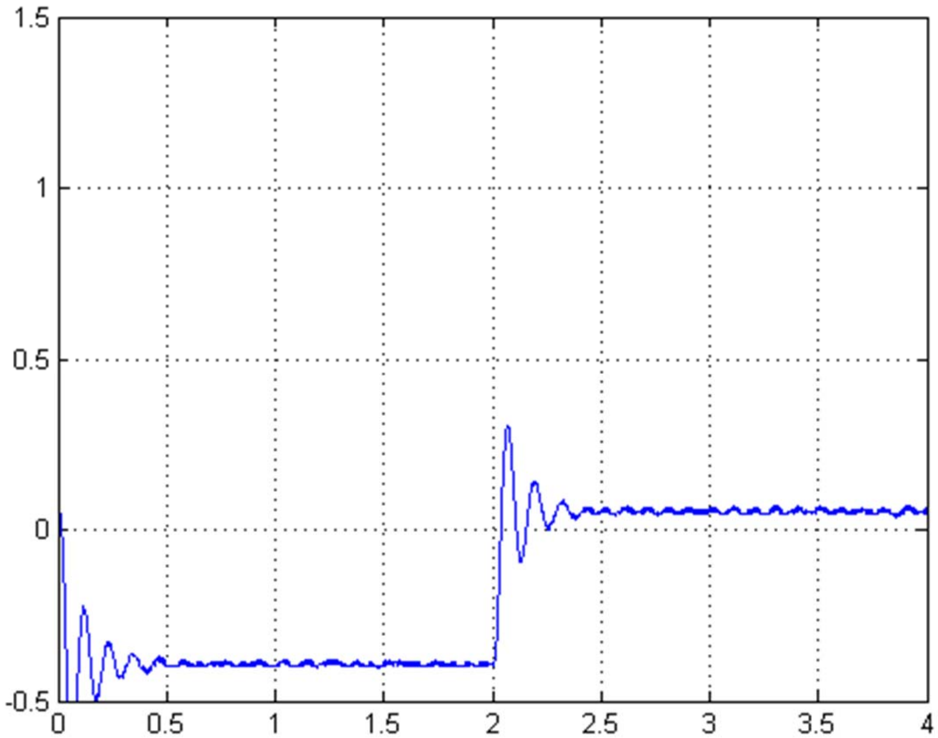
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ w \end{bmatrix}$$

$$w = k_1(ref - x_1) - k_2 x_2$$

$$u = \left(\sqrt{(\gamma g - w) m (x_1/\gamma + X_0)^2 / (\gamma k)} - I_0 \right) / 0.15$$

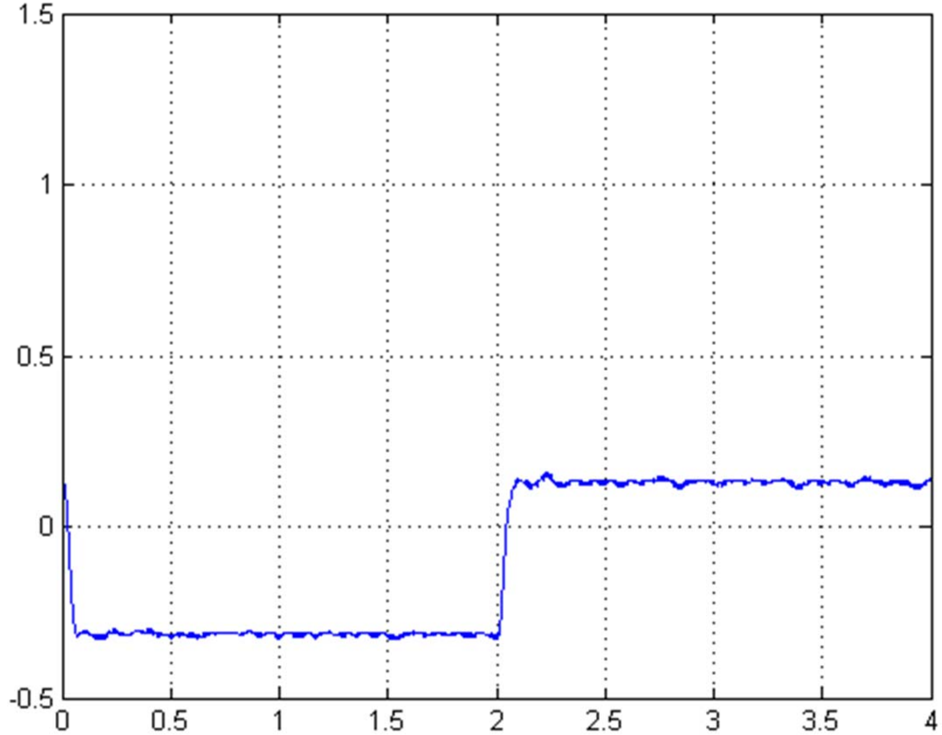
Input-Output Linearization

$$Poles: -20 \pm j50$$



Input-Output Linearization

$$Poles: -50, -50$$



Exercise 1

- Lead compensator에 대한 제어기를 실행하여 응답을 관찰하시오.
- 세가지 lead compensator에 대해서, Bode plot을 그리고, step response의 simulation을 실행하여, phase margin의 변화에 따라서 응답이 어떻게 달라지는지 관찰하시오. 또한 simulation과 실험 결과를 비교해 보시오.

Exercise 1

- Observe the step responses with lead compensators.
- For the three lead compensators, plot Bode plots and step response simulations. Discuss how the phase margins affects the transients of responses. Also compare the step response simulations with the experiments.

Exercise 2

- Input-output linearization method를 이용한 제어기를 주어진 소스 코드를 활용하여 구현하시오. Linear controller는 closed-loop pole이 다음과 같은 state feedback controller로 구성하시오.

$$Poles: -20 \pm j50$$

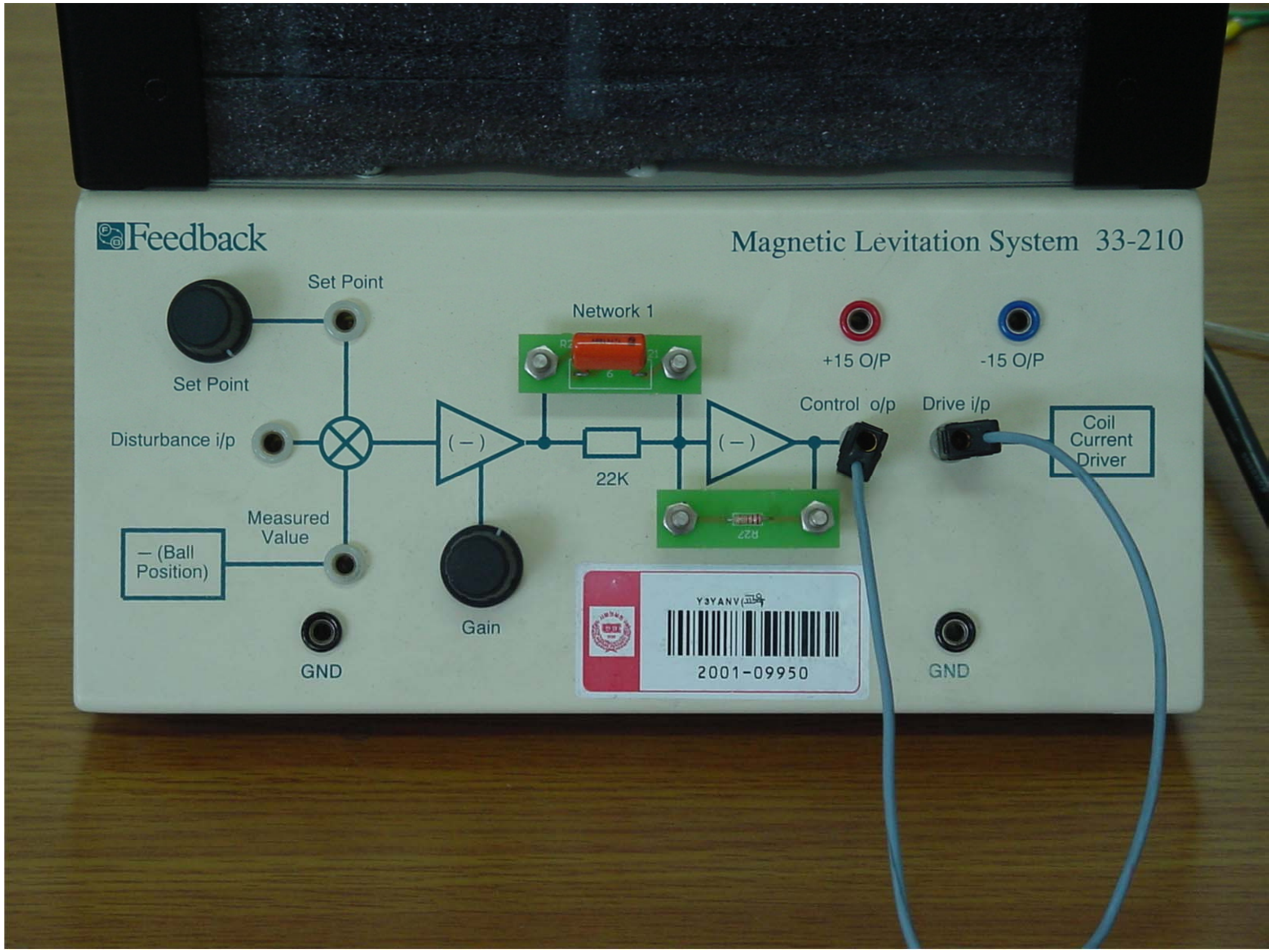
$$Poles: -50, -50$$

Exercise 2

- Implement the state feedback controller using the input-output linearization method. The closed-loop poles of the linear state feedback controller are as follows:

$$Poles: -20 \pm j50$$

$$Poles: -50, -50$$


Feedback
Magnetic Levitation System 33-210
Set Point
Network 1
+15 O/P
-15 O/P
Set Point
Control o/p
Drive i/p
Disturbance i/p
(–)
(–)
Coil
Current
Driver
22K
Measured
Value
– (Ball
Position)
Gain
GND
2001-09950
GND

Ball & Beam

IEEE Transactions on Automatic Control, March 1992

Nonlinear Control Via Approximate Input–Output Linearization: The Ball and Beam Example

John Hauser, Shankar Sastry, and Petar Kokotović

Abstract—**We study approximate input–output linearization of nonlinear systems which fail to have a well defined relative degree. For such systems, we provide a method for constructing approximate systems that are input–output linearizable. The analysis presented in this note is motivated through its application to a common undergraduate control laboratory experiment—the ball and beam system—where it is shown to be more effective for trajectory tracking than the standard Jacobian linearization.**

Ball & Beam

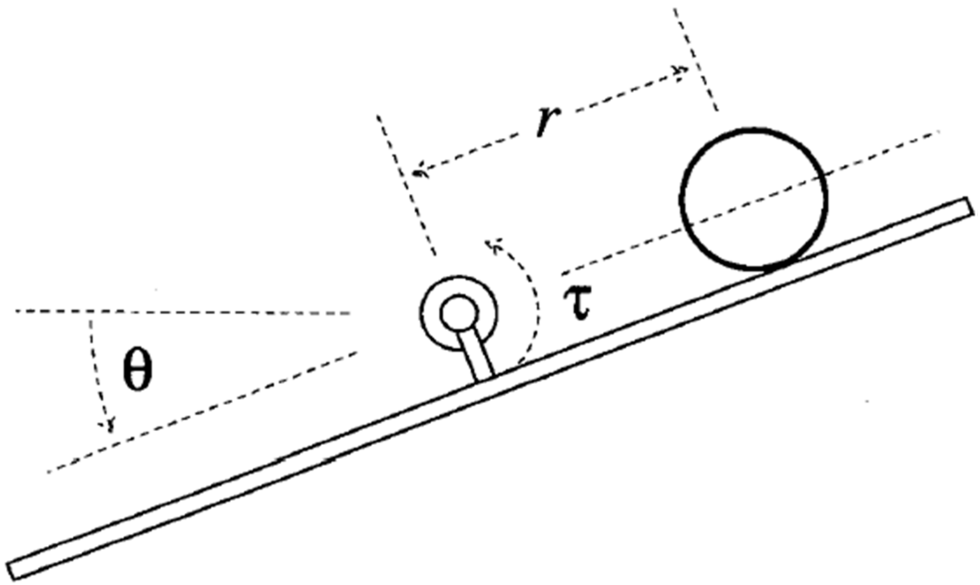


Fig. 1. The ball and beam system.

$$0 = \left(\frac{J_b}{R^2} + M\right)\ddot{r} + MG\sin\theta - Mr\dot{\theta}^2$$

$$\tau = \left(Mr^2 + J + J_b\right)\ddot{\theta} + 2Mr\dot{r}\dot{\theta} + MGr\cos\theta$$

State Equation

$$B = \frac{M}{\left(\frac{J_b}{R^2} + M\right)}$$

$$\tau = 2Mr\dot{r}\dot{\theta} + MGr\cos\theta + \left(Mr^2 + J + J_b\right)u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \underbrace{\begin{bmatrix} x_2 \\ B\left(x_1 x_4^2 - G\sin x_3\right) \\ x_4 \\ 0 \end{bmatrix}}_{f(x)} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}_{g(x)} u$$

$$y = \underbrace{x_1}_{h(x)}$$

where $x = (x_1, x_2, x_3, x_4)^T := (r, \dot{r}, \theta, \dot{\theta})^T$ and $y = h(x) := r$.

Input-Output Linearization

$$y = x_1,$$

$$\dot{y} = x_2,$$

$$\ddot{y} = Bx_1 x_4^2 - BG \sin x_3,$$

$$y^{(3)} = \underbrace{Bx_2 x_4^2 - BGx_4 \cos x_3}_{b(x)} + \underbrace{2Bx_1 x_4}_{a(x)} u.$$

$$u = (-b(x) + v)/a(x) \qquad y^{(3)} = v.$$

- Relative degree is not well defined!

Input-State Linearization

- Integrability condition is not satisfied.

$$\text{span}\left\{g, ad_f g, \cdots, ad_f^{n-2} g\right\} \qquad (2.5)$$

since $[g, ad_f^2 g] = (2Bx_1 \;\; -2Bx_2 \;\; 0 \;\; 0)^T$ does not lie in (2.5). Here $ad_f^i g$ denotes the iterated *Lie bracket* $[f, \cdots [f, g] \cdots]$. Thus, it is not possible to fully linearize the ball and beam system.

Approximate Input-Output Linearization: 1

Let $\xi_1 = \phi_1(x) = h(x)$.

$$\dot{\xi}_1 = \underbrace{x_2}_{\xi_2 = \phi_2(x)}$$

$$\dot{\xi}_2 = \underbrace{-BG \sin x_3}_{\xi_3 = \phi_3(x)} + \underbrace{B x_1 x_4^2}_{\psi_2(x)}$$

$$\dot{\xi}_3 = \underbrace{-BG x_4 \cos x_3}_{\xi_4 = \phi_4(x)}$$

$$\dot{\xi}_4 = \underbrace{BG x_4^2 \sin x_3}_{b(x)} + \underbrace{(-BG \cos x_3)}_{a(x)} u$$

or

$$\dot{\xi}_1 = \xi_2$$

$$\dot{\xi}_2 = \xi_3 + \psi_2(x)$$

$$\dot{\xi}_3 = \xi_4$$

$$\dot{\xi}_4 = b(x) + a(x) u$$

$$=: v(x, u). \tag{3.2}$$

Approximate Input-Output Linearization: 2

Let $\xi_1 = \phi_1(x) = h(x)$.

$$\dot{\xi}_1 = \underbrace{x_2}_{\xi_2 = \phi_2(x)}$$

$$\dot{\xi}_2 = \underbrace{-BG\sin x_3 + Bx_1x_4^2}_{\xi_3 = \phi_3(x)}$$

$$\dot{\xi}_3 = \underbrace{-BGx_4\cos x_3 + Bx_2x_4^2}_{\xi_4 = \phi_4(x)} + \underbrace{2Bx_1x_4u}_{\psi_3(x,u)}$$

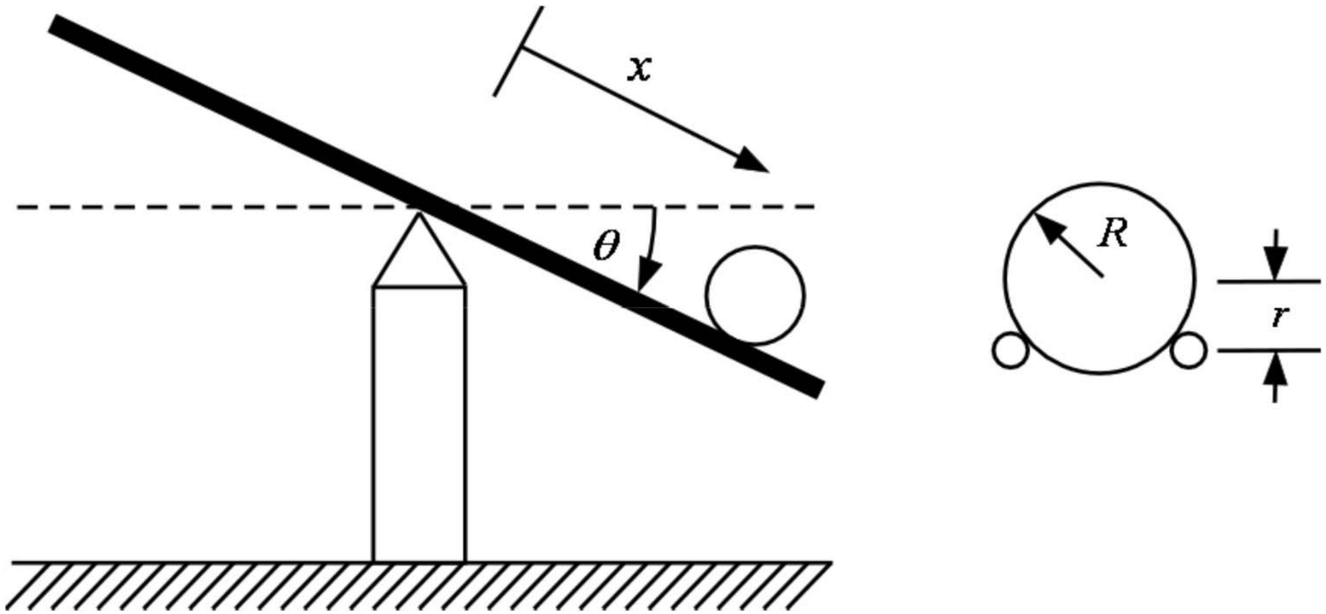
$$\dot{\xi}_4 = \underbrace{B^2x_1x_4^4 + B(1-B)x_4^2\sin x_3}_{b(x)} + \underbrace{(-BG\cos x_3 + 2Bx_2x_4)}_{a(x)}u$$

or

$$\begin{aligned}
\dot{\xi}_1 &= \xi_2 \\
\dot{\xi}_2 &= \xi_3 \\
\dot{\xi}_3 &= \xi_4 + \psi_3(x,u) \\
\dot{\xi}_4 &= b(x) + a(x)u \\
&=: v(x,u).
\end{aligned}$$

$$\begin{aligned}
v = y_d^{(4)}(t) &+ \alpha_3\left(y_d^{(3)}(t) - \phi_4(x)\right) + \alpha_2\left(\ddot{y}_d(t) - \phi_3(x)\right) \\
&+ \alpha_1\left(\dot{y}_d(t) - \phi_2(x)\right) + \alpha_0\left(y_d(t) - \phi_1(x)\right) \quad (3.1)
\end{aligned}$$

Ball & Beam



Dynamic Equation

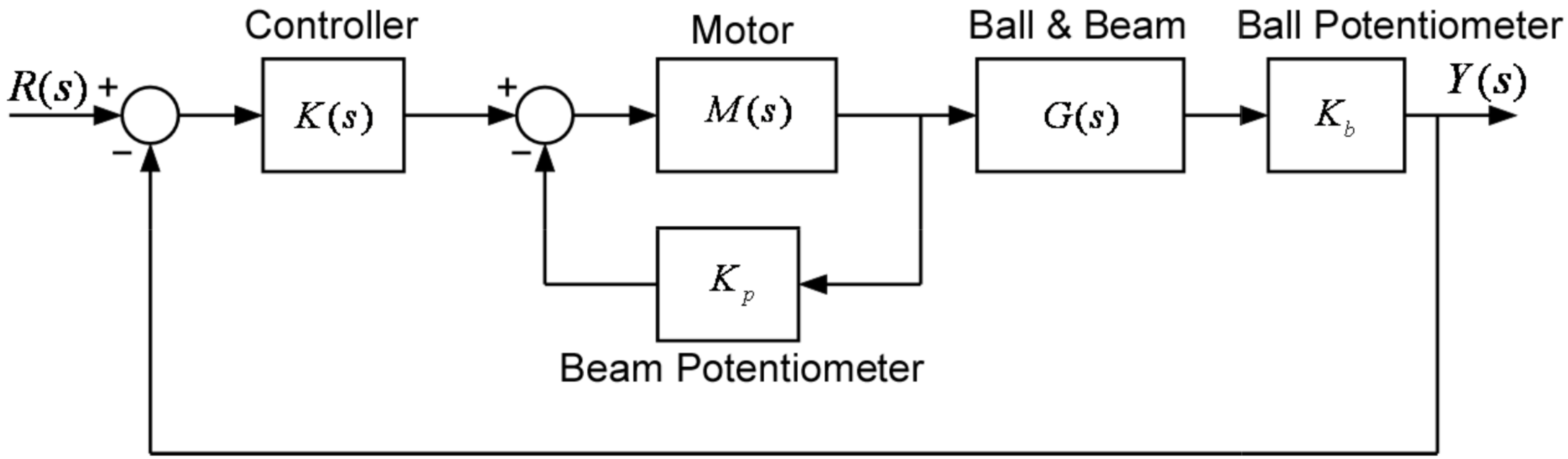
$$\left(m + \frac{I_b}{r^2}\right)\ddot{x} + \left(\frac{I_b}{r^2}\right)\ddot{\theta} - mx\dot{\theta}^2 = mg\sin\theta$$

Assume $\dot{\theta} \approx 0, \ddot{\theta} \approx 0, \sin\theta \approx \theta$

$$\left(m + \frac{I_b}{r^2}\right)\ddot{x} = mg\theta, I_b = \frac{2}{5}mR^2$$

$$\frac{X(s)}{\Theta(s)} = \frac{mg}{\left(m + \frac{I_b}{r^2}\right)s^2} = \frac{g}{\left(1 + \frac{2}{5}\frac{R^2}{r^2}\right)s^2}$$

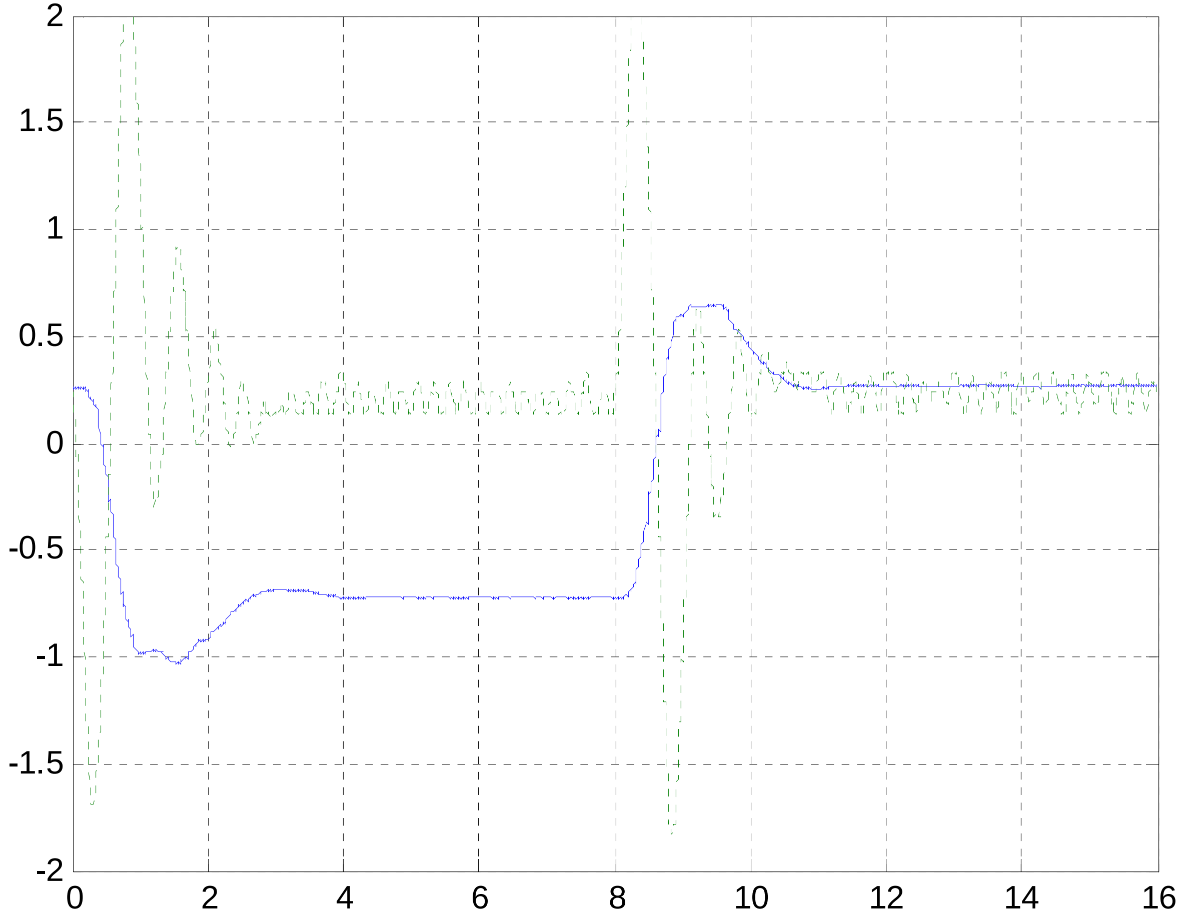
Control System



$$\frac{M(s)}{1+K_pM(s)} \approx \frac{M(0)}{1+K_pM(0)}$$

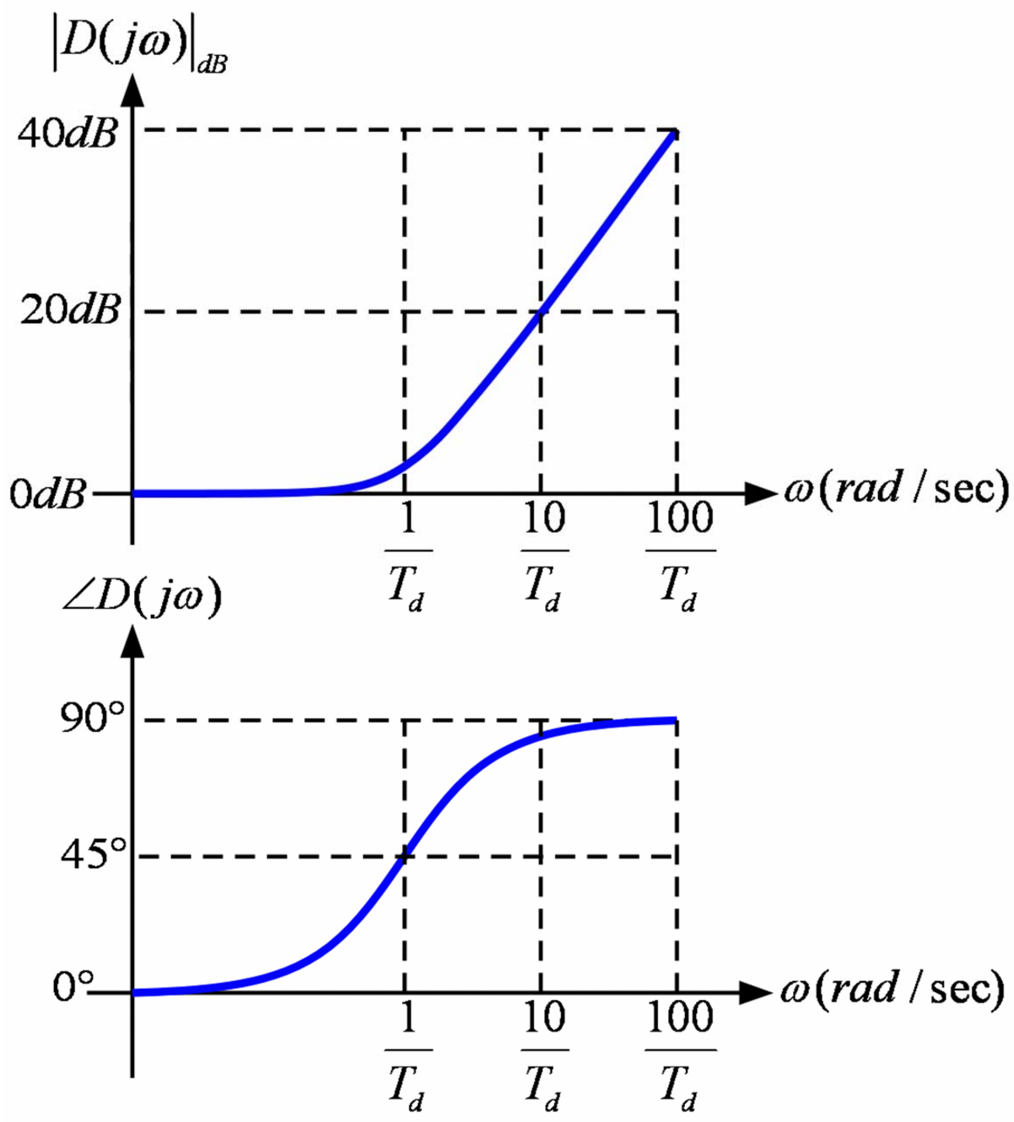
Exercise

- Try with PD controller.
- Design and implement two different lead compensator with K=1 and phase margin=30~50 degrees.
- Compare experimental results with simulations.


2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
2
4
6
8
10
12
14
16

PD 제어기의 설계

$$D(s) = K\left(1 + T_d s\right)$$



예제 7-1

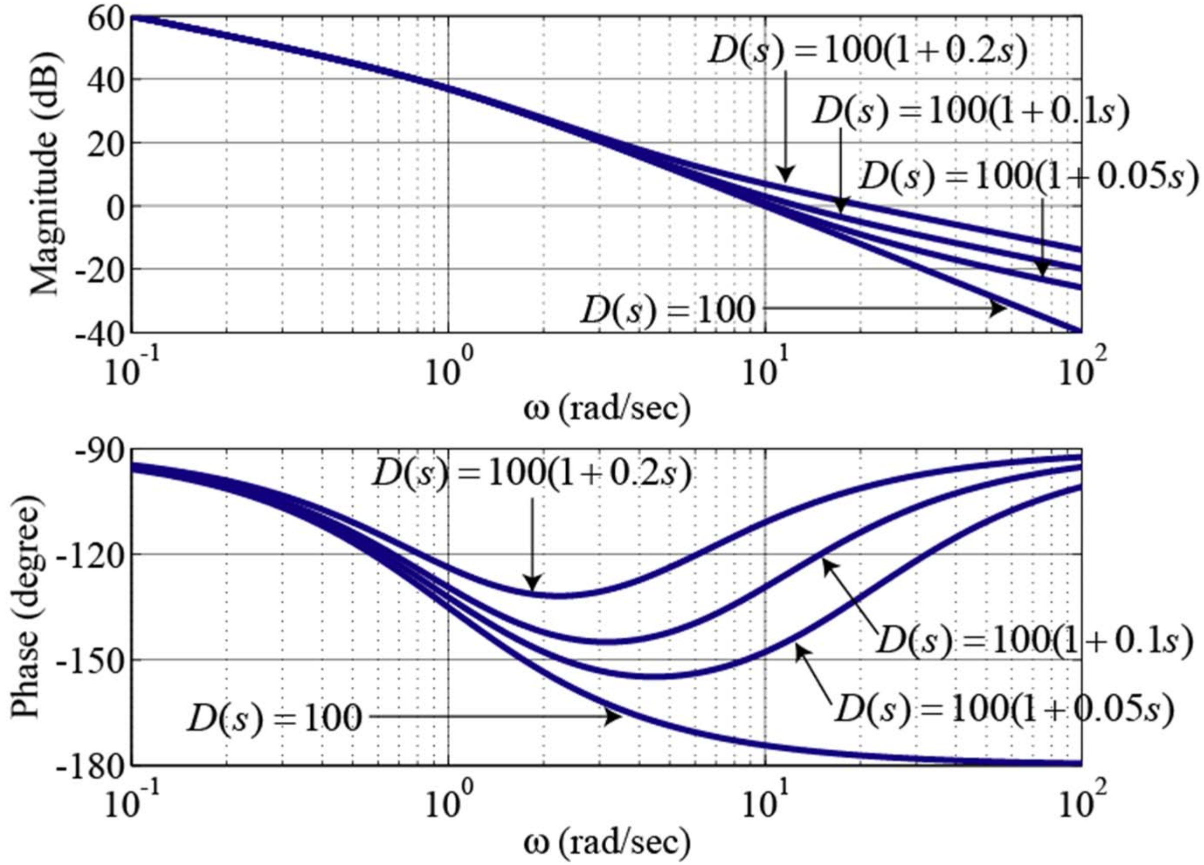
$$G(s)=\frac{1}{s(s+1)}$$

$$K_v=\lim_{s\to 0} s\frac{K}{s(s+1)}=K$$

$$e_{ss}=\frac{1}{K_v}=\frac{1}{K}$$

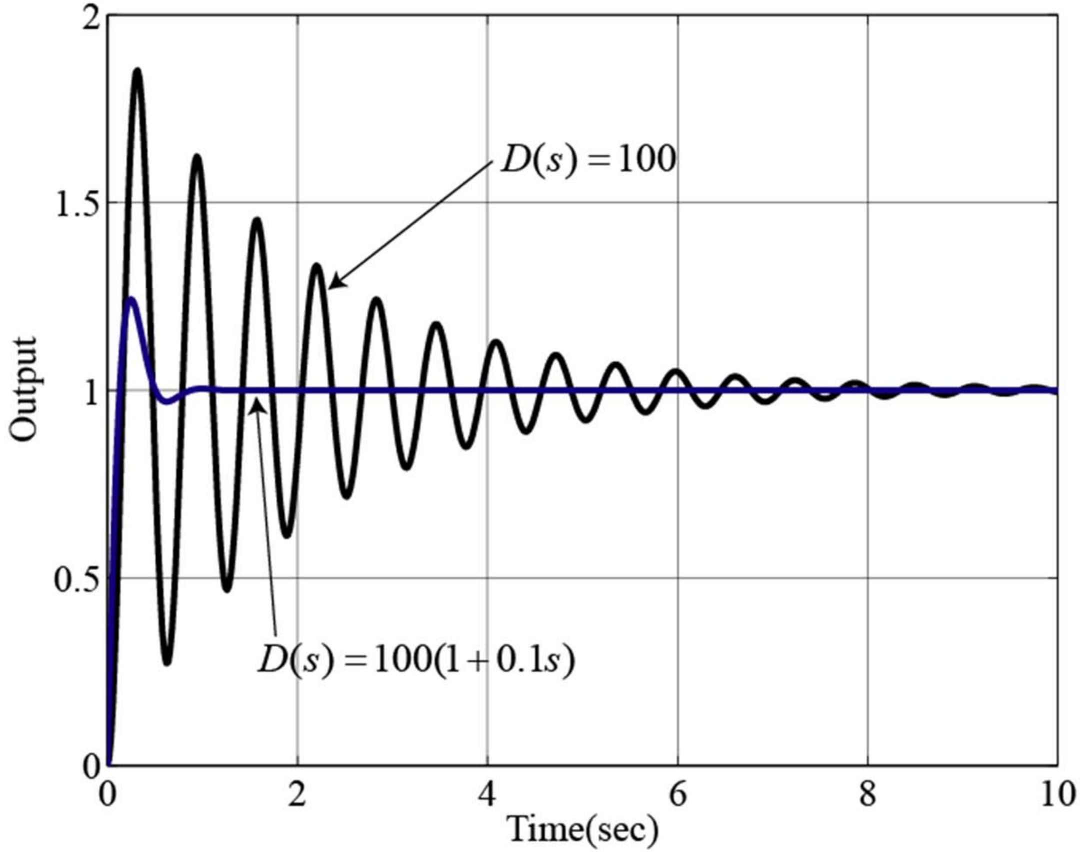
$$D(s)=100(1+0.1s)$$

예제 7-1



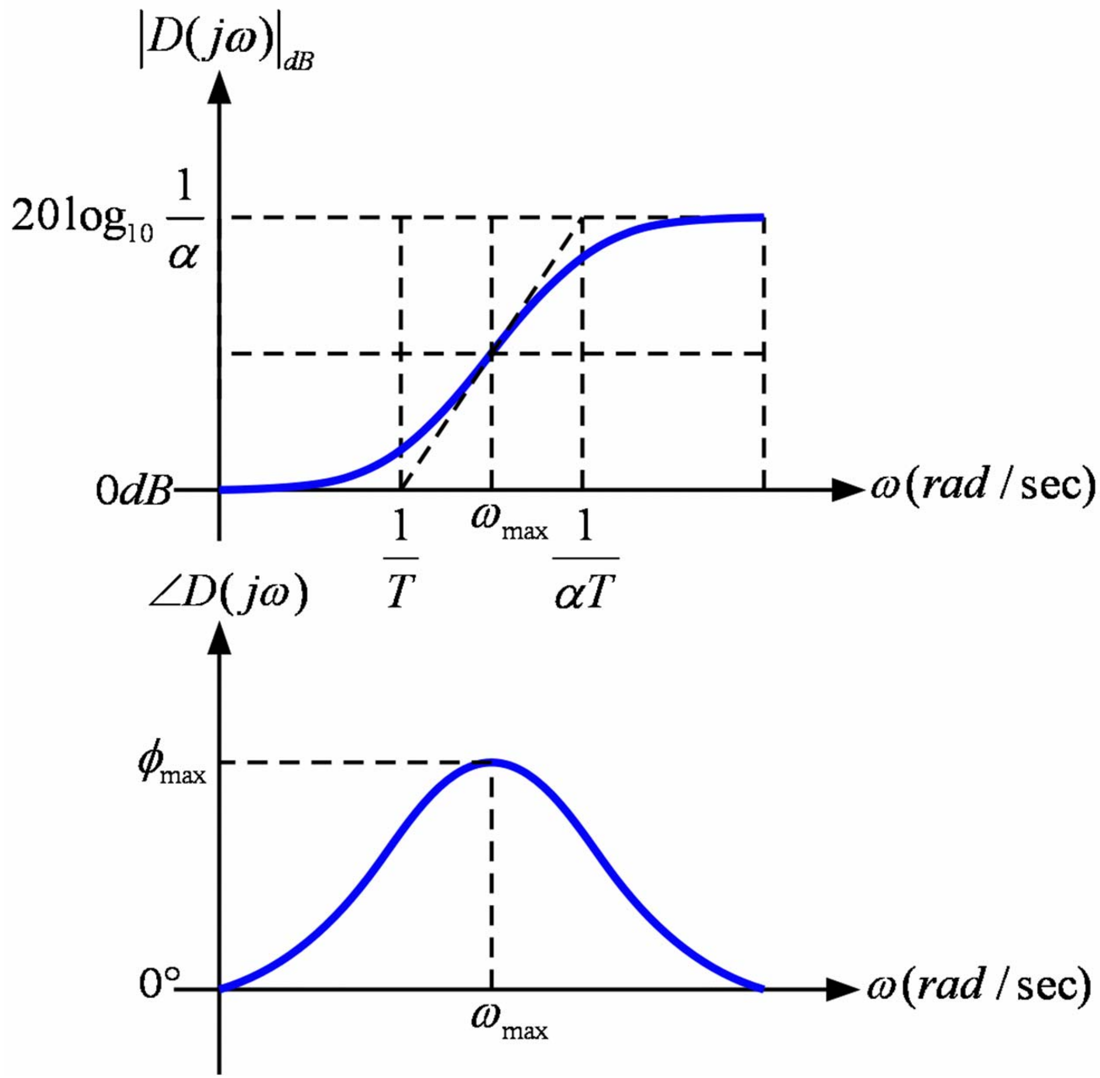
60
40
20
0
-20
-40
Magnitude (dB)
D(s) = 100(1 + 0.2s)
D(s) = 100(1 + 0.1s)
D(s) = 100(1 + 0.05s)
D(s) = 100
10^-1
10^0
10^1
10^2
ω (rad/sec)
-90
-120
-150
-180
Phase (degree)
D(s) = 100(1 + 0.2s)
D(s) = 100(1 + 0.1s)
D(s) = 100(1 + 0.05s)
D(s) = 100
10^-1
10^0
10^1
10^2
ω (rad/sec)

예제 7-1



진상 제어기의 설계

$$D(s) = K\frac{Ts+1}{\alpha Ts+1}$$



진상 제어기의 설계

$$\phi = \angle\left(\frac{jT\omega+1}{j\alpha T\omega+1}\right) = \tan^{-1}(T\omega) - \tan^{-1}(\alpha T\omega)$$

$$\log_{10}\omega_{max} = \frac{1}{2}\left(\log_{10}\frac{1}{T} + \log_{10}\frac{1}{\alpha T}\right) \qquad \omega_{max} = \frac{1}{T\sqrt{\alpha}}$$

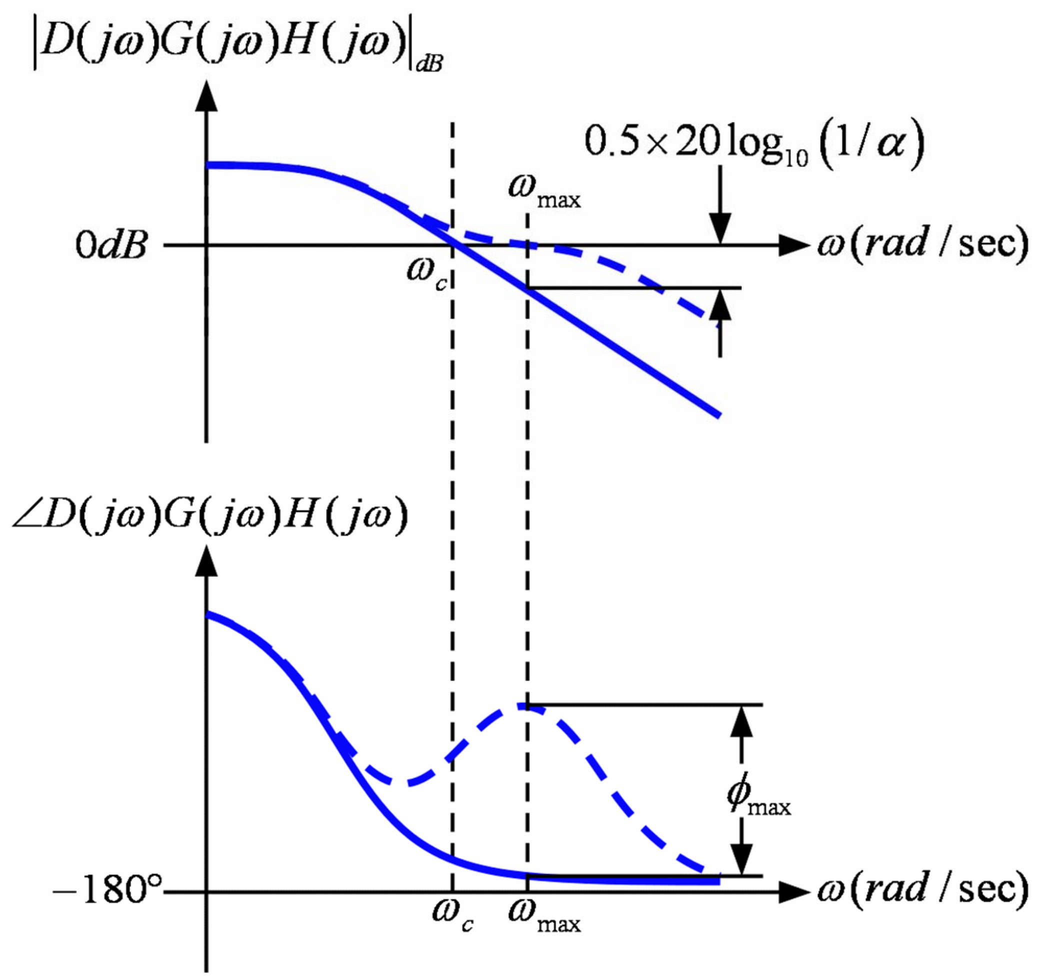
$$\phi_{max} = \tan^{-1}\frac{1}{\sqrt{\alpha}} - \tan^{-1}\sqrt{\alpha}$$

$$\tan\phi_{max} = \frac{1-\alpha}{2\sqrt{\alpha}} \qquad \sin\phi_{max} = \frac{1-\alpha}{1+\alpha} \qquad \alpha = \frac{1-\sin\phi_{max}}{1+\sin\phi_{max}}$$

진상 제어기의 설계

$$20\log_{10}\left|KG(j\omega_{\max})H(j\omega_{\max})\right| = -0.5\times 20\log_{10}\frac{1}{\alpha}$$

$$T = \frac{1}{\omega_{\max}\sqrt{\alpha}}$$



예제 7-2

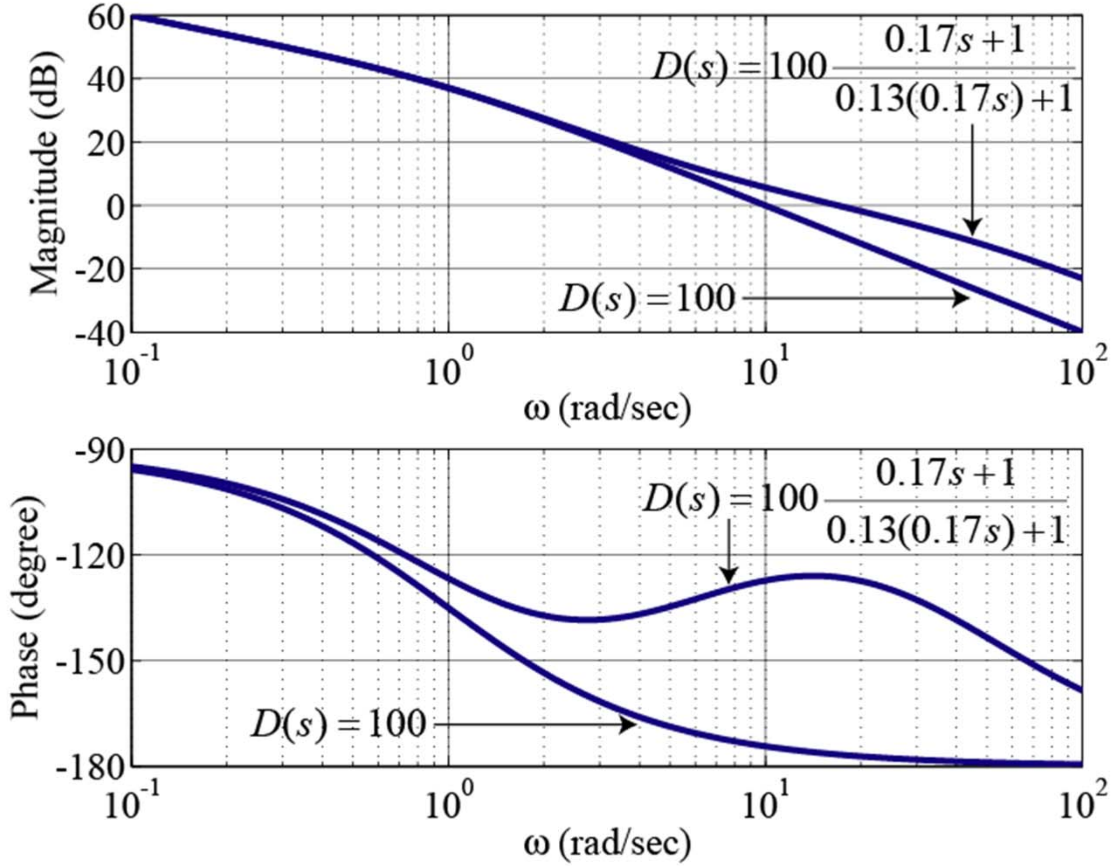
$$\phi_{\max} = 50^\circ \qquad \alpha = 0.13$$

$$0.5 \times 20\log_{10}\left(1/\alpha\right) = 9dB$$

$$T = \frac{1}{\omega_{\max}\sqrt{\alpha}} = \frac{1}{16.7\sqrt{0.13}} = 0.17$$

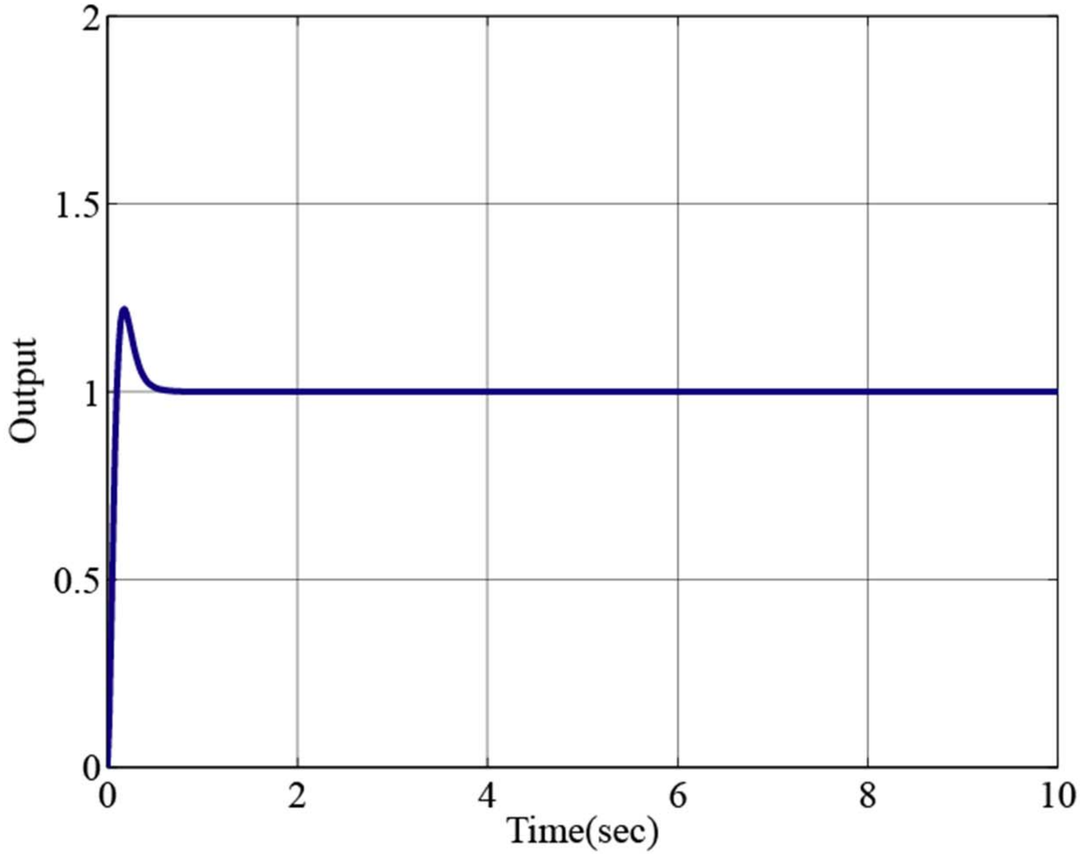
$$D(s) = K\frac{Ts+1}{\alpha Ts+1} = 100\frac{0.17s+1}{0.13(0.17s)+1}$$

예제 7-2



60
40
20
0
-20
-40
Magnitude (dB)
$D(s)=100\frac{0.17s+1}{0.13(0.17s)+1}$
$D(s)=100$
10^{-1}
10^{0}
10^{1}
10^{2}
ω (rad/sec)
-90
-120
-150
-180
Phase (degree)
$D(s)=100\frac{0.17s+1}{0.13(0.17s)+1}$
$D(s)=100$
10^{-1}
10^{0}
10^{1}
10^{2}
ω (rad/sec)

예제 7-2



Convert to Digital Form

$$D(s) = \frac{U(s)}{E(s)} = K\,\frac{Ts+1}{\alpha Ts+1}$$

$$D(z) = \frac{U(z)}{E(z)} = K\,\frac{Ts+1}{\alpha Ts+1}\Bigg|_{s=\frac{2}{T_s}\frac{z-1}{z+1}} = K\,\frac{T\,\frac{2}{T_s}\,\frac{z-1}{z+1}+1}{\alpha T\,\frac{2}{T_s}\,\frac{z-1}{z+1}+1}$$

$$u(n) = \frac{1}{T_s+2\alpha T}\Big[K\big(T_s+2T\big)e(n)+K\big(T_s-2T\big)e(n-1)+\big(2\alpha T-T_s\big)u(n-1)\Big]$$