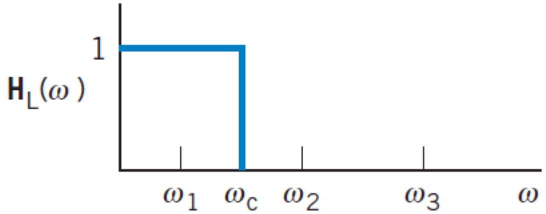
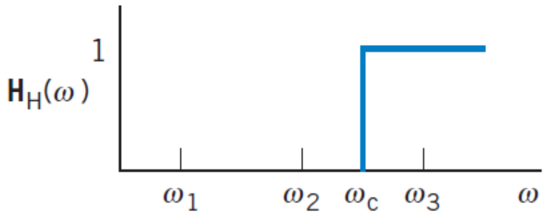
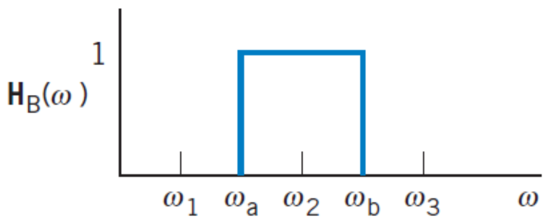
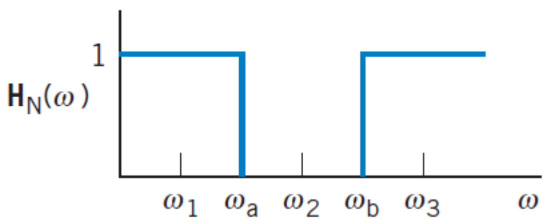
Active Filter

High-Pass Filter

Band-Pass Filter

Band-Reject Filter

Ideal Filters

FILTER TYPE	IDEAL FREQUENCY RESPONSE	FILTER INPUT AND OUTPUT
Low-pass		$v_i(t) = \cos\omega_1 t + \cos\omega_2 t + \cos\omega_3 t$ → $\mathbf{H}_L(\omega)$ → $v_o(t) = \cos\omega_1 t$
High-pass		$v_i(t) = \cos\omega_1 t + \cos\omega_2 t + \cos\omega_3 t$ → $\mathbf{H}_H(\omega)$ → $v_o(t) = \cos\omega_3 t$
Band-pass		$v_i(t) = \cos\omega_1 t + \cos\omega_2 t + \cos\omega_3 t$ → $\mathbf{H}_B(\omega)$ → $v_o(t) = \cos\omega_2 t$
Band-stop (notch)		$v_i(t) = \cos\omega_1 t + \cos\omega_2 t + \cos\omega_3 t$ → $\mathbf{H}_N(\omega)$ → $v_o(t) = \cos\omega_1 t + \cos\omega_3 t$

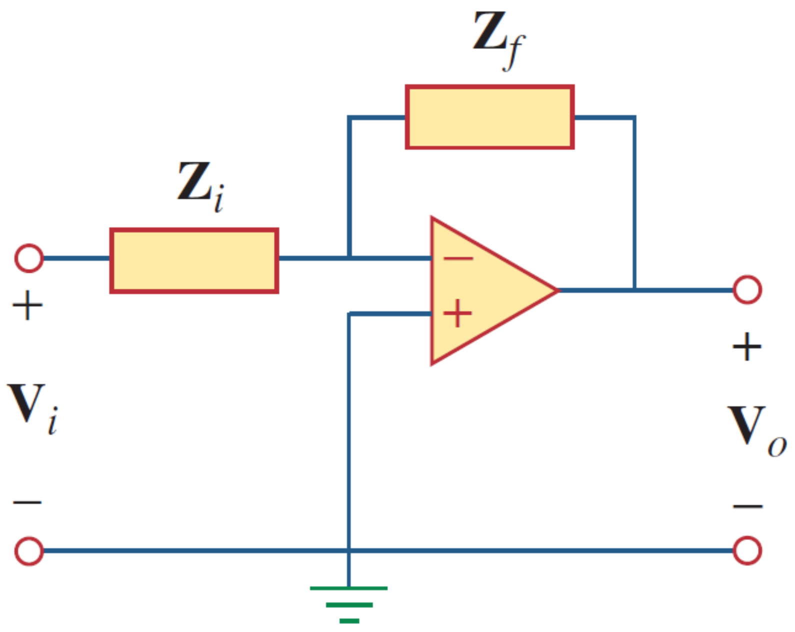
Passive Filter Limitations

- They cannot generate gain greater than 1: passive elements cannot add energy to the network
- They may require bulky and expensive inductors
- Loading effects: frequency response may change depending on the impedances of the load

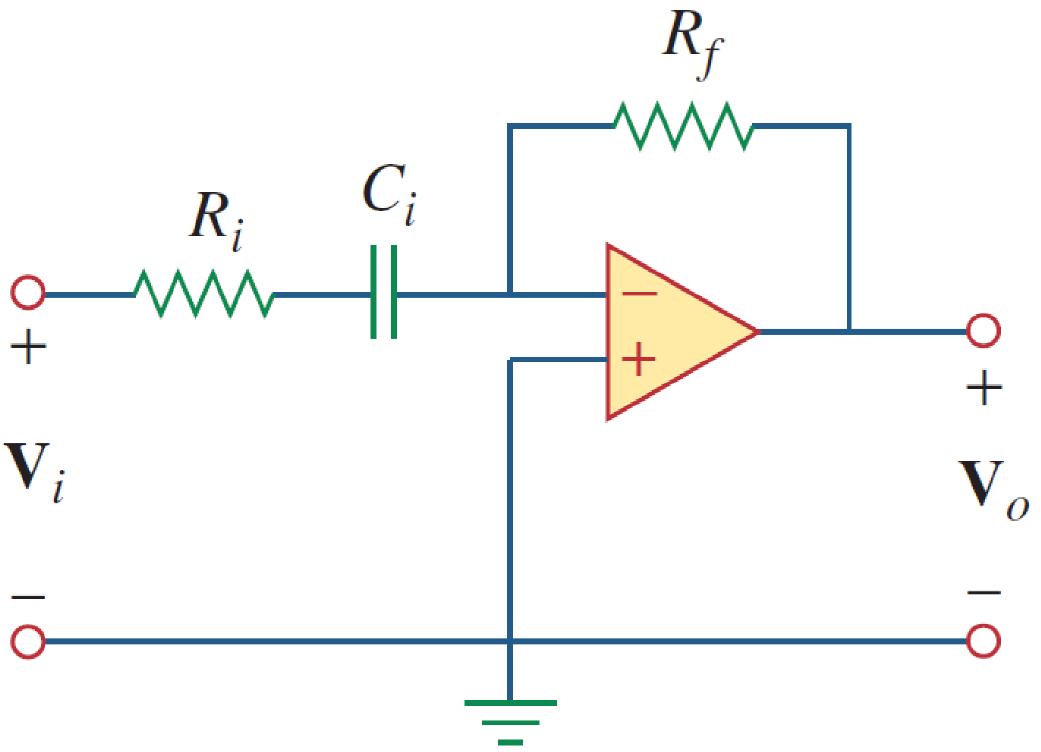
Active filters

- They are often smaller and less expensive, because they do not require inductors. This makes feasible the integrated circuit realizations of filters.
- They can provide amplifier gain
- Active filters can be combined with buffer amplifiers (voltage followers) to isolate each stage of the filter from source and load impedance effects.

First-Order High-Pass Filter



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = -\frac{\mathbf{Z}_f}{\mathbf{Z}_i}$$



R_f
C_i
R_i
+
$\mathbf{V}_i$
−
+
$\mathbf{V}_o$
−

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = -\frac{\mathbf{Z}_f}{\mathbf{Z}_i}$$

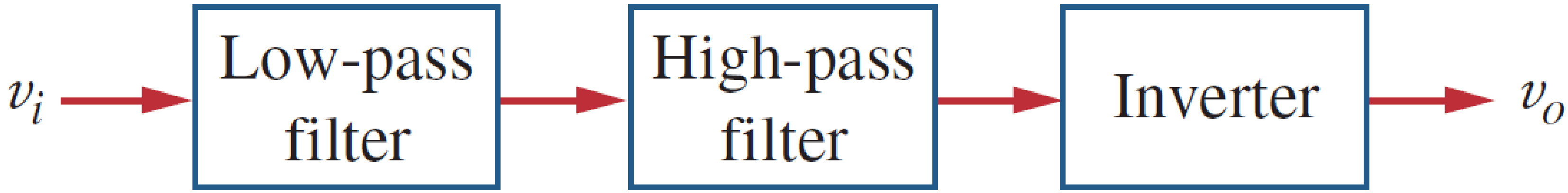
where $\mathbf{Z}_i = R_i + 1/j\omega C_i$ and $\mathbf{Z}_f = R_f$ so that

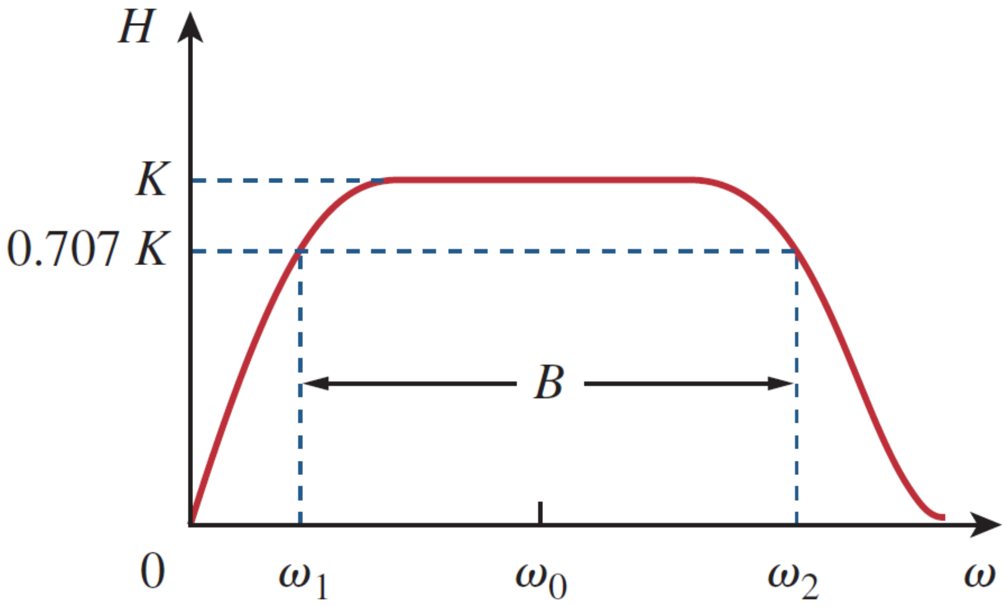
$$\mathbf{H}(\omega) = -\frac{R_f}{R_i + 1/j\omega C_i} = -\frac{j\omega C_i R_f}{1 + j\omega C_i R_i}$$

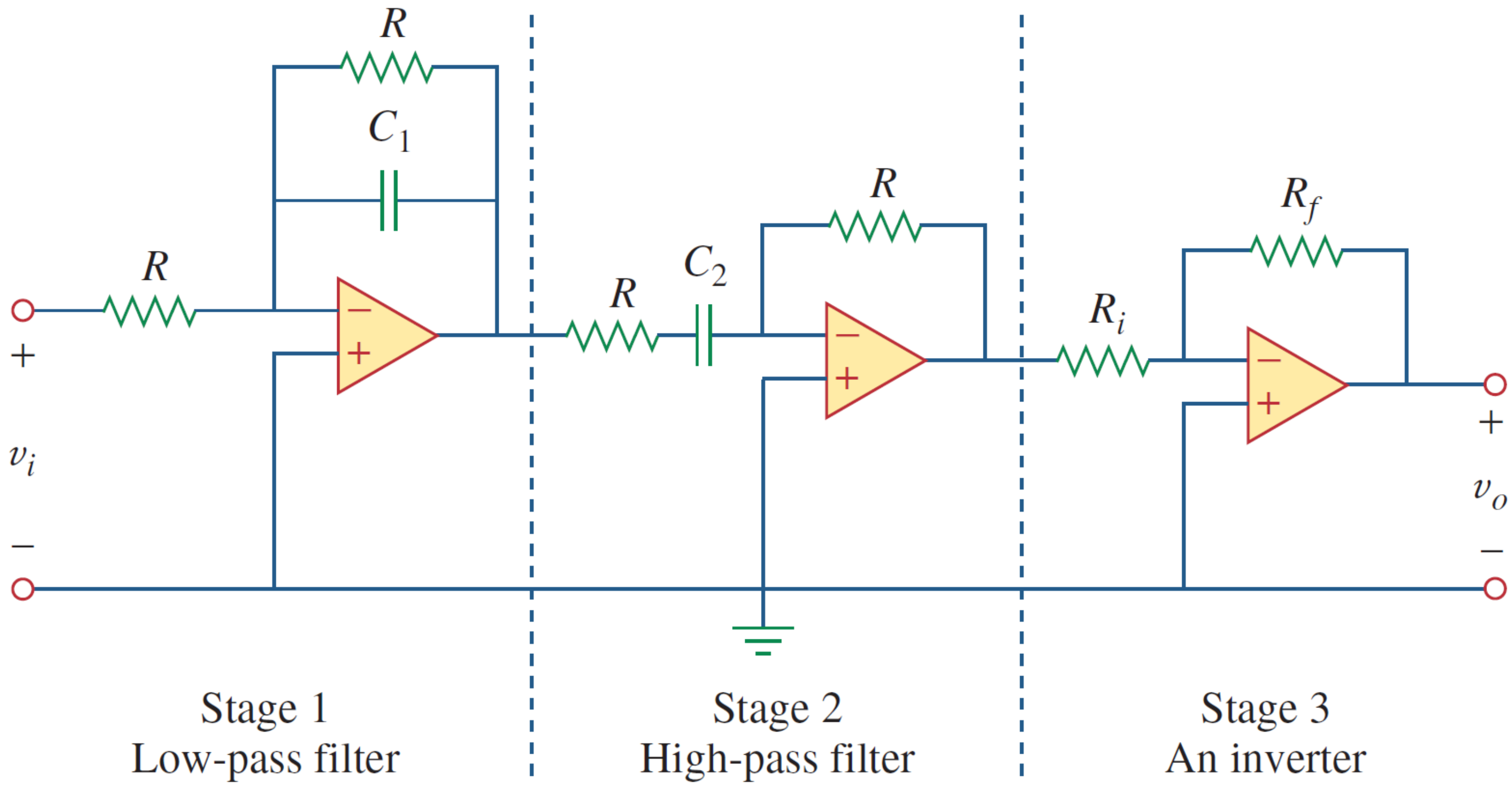
$(\omega \rightarrow \infty)$, the gain tends to $-R_f/R_i$. The corner frequency is

$$\omega_c = \frac{1}{R_i C_i}$$

Band-Pass Filter







R
C1
R
C2
R
Rf
R
Ri
+
vi
−
+
vo
−
Stage 1
Low-pass filter
Stage 2
High-pass filter
Stage 3
An inverter

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \left(-\frac{1}{1 + j\omega C_1 R}\right)\left(-\frac{j\omega C_2 R}{1 + j\omega C_2 R}\right)\left(-\frac{R_f}{R_i}\right)$$

$$= -\frac{R_f}{R_i}\frac{1}{1 + j\omega C_1 R}\frac{j\omega C_2 R}{1 + j\omega C_2 R}$$

The lowpass section sets the upper corner frequency as

$$\omega_2 = \frac{1}{RC_1}$$

while the highpass section sets the lower corner frequency as

$$\omega_1 = \frac{1}{RC_2}$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

$$B = \omega_2 - \omega_1$$

$$Q = \frac{\omega_0}{B}$$

$$\mathbf{H}(\omega) = -\frac{R_f}{R_i} \frac{j\omega/\omega_1}{(1 + j\omega/\omega_1)(1 + j\omega/\omega_2)} = -\frac{R_f}{R_i} \frac{j\omega\omega_2}{(\omega_1 + j\omega)(\omega_2 + j\omega)}$$

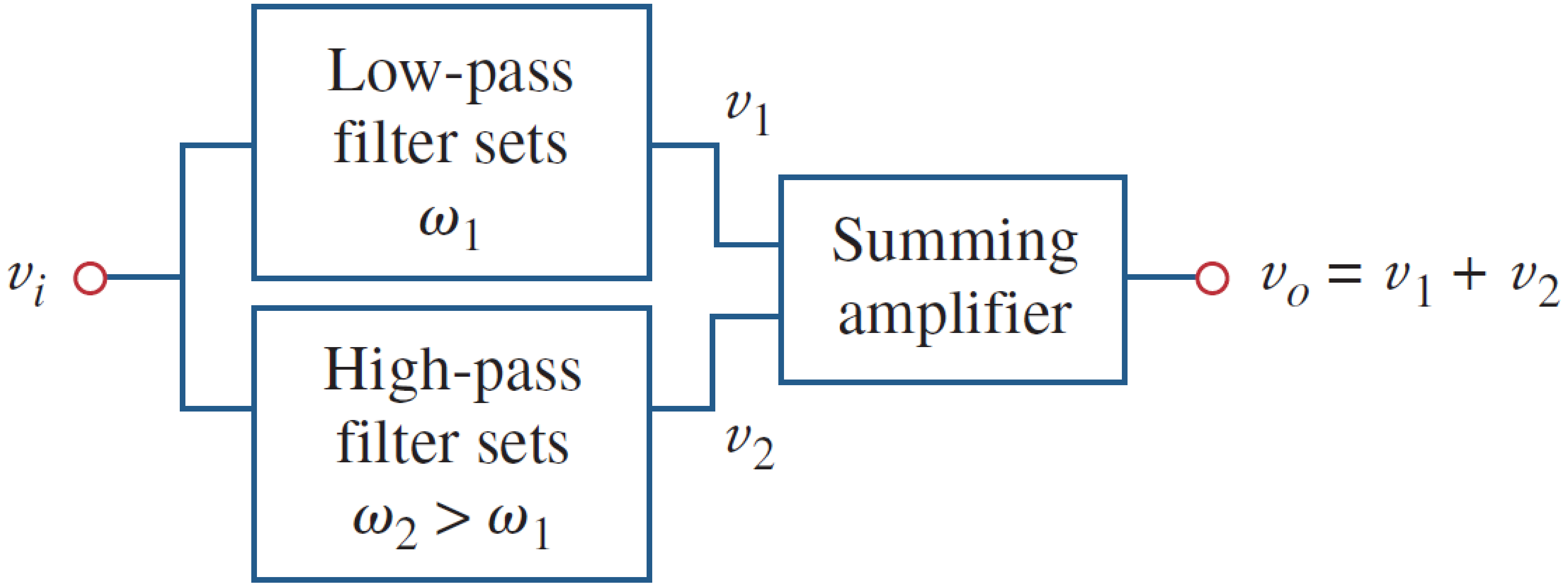
At the center frequency $\omega_0 = \sqrt{\omega_1 \omega_2}$, the magnitude of the transfer function is

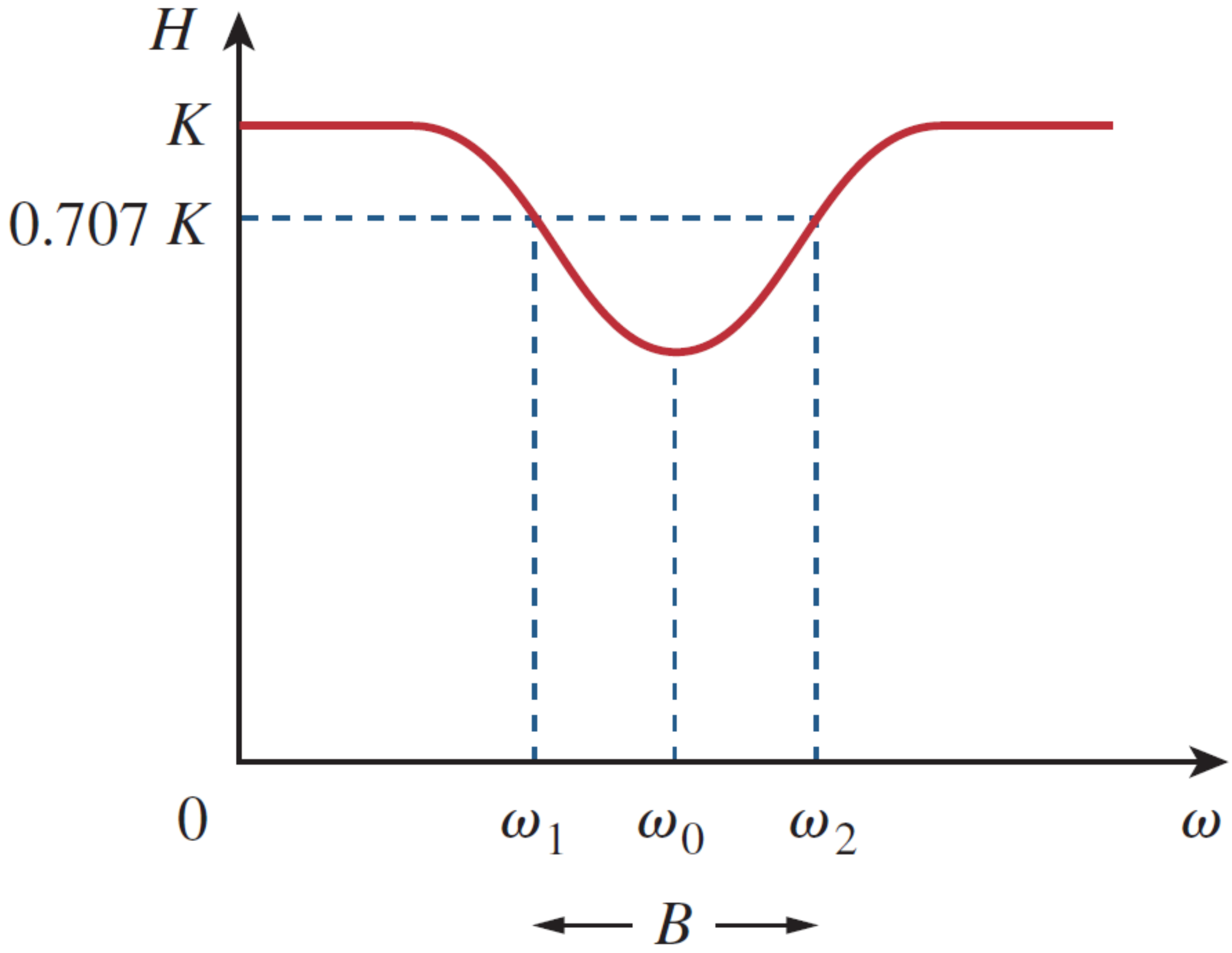
$$|\mathbf{H}(\omega_0)| = \left| \frac{R_f}{R_i} \frac{j\omega_0 \omega_2}{(\omega_1 + j\omega_0)(\omega_2 + j\omega_0)} \right| = \frac{R_f}{R_i} \frac{\omega_2}{\omega_1 + \omega_2} \quad \textbf{(14.72)}$$

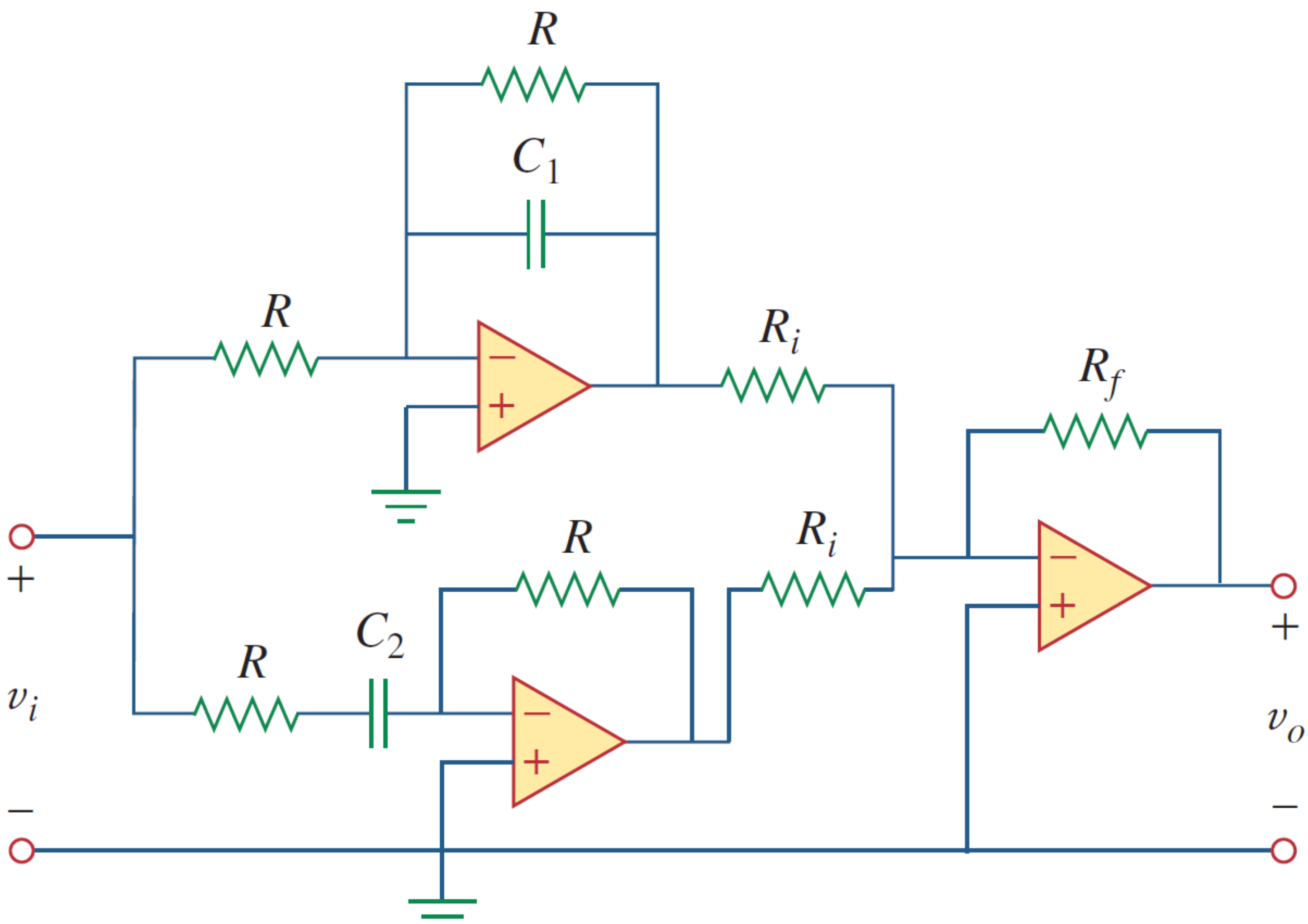
Thus, the passband gain is

$$K = \frac{R_f}{R_i} \frac{\omega_2}{\omega_1 + \omega_2} \quad \textbf{(14.73)}$$

Band-Reject (Notch) Filter




H
K
0.707 K
0
ω1
ω0
ω2
ω
B


R
C1
R
Ri
Rf
R
Ri
+
C2
R
vi
−
+
vo
−

$$\mathbf{H}(\omega) = \frac{R_f}{R_i}\left(\frac{1}{1 + j\omega/\omega_2} + \frac{j\omega/\omega_1}{1 + j\omega/\omega_1}\right)$$

$$= \frac{R_f}{R_i}\,\frac{(1 + j2\omega/\omega_1 + (j\omega)^2/\omega_1\omega_1)}{(1 + j\omega/\omega_2)(1 + j\omega/\omega_1)}$$

$(\omega \rightarrow 0$ and $\omega \rightarrow \infty)$ the gain is

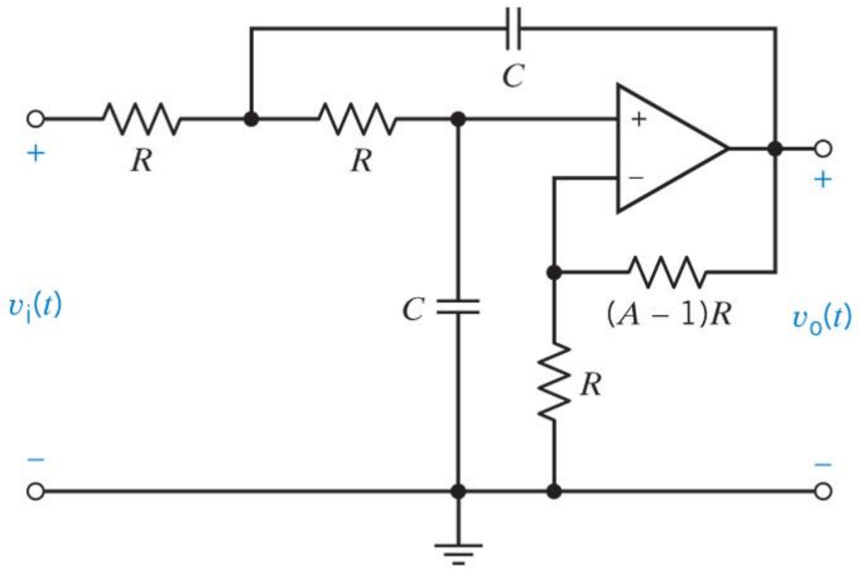
$$K = \frac{R_f}{R_i}$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

$$H(\omega_0) = \left| \frac{R_f}{R_i} \frac{(1 + j2\omega_0/\omega_1 + (j\omega_0)^2/\omega_1\omega_1)}{(1 + j\omega_0/\omega_2)(1 + j\omega_0/\omega_1)} \right|$$

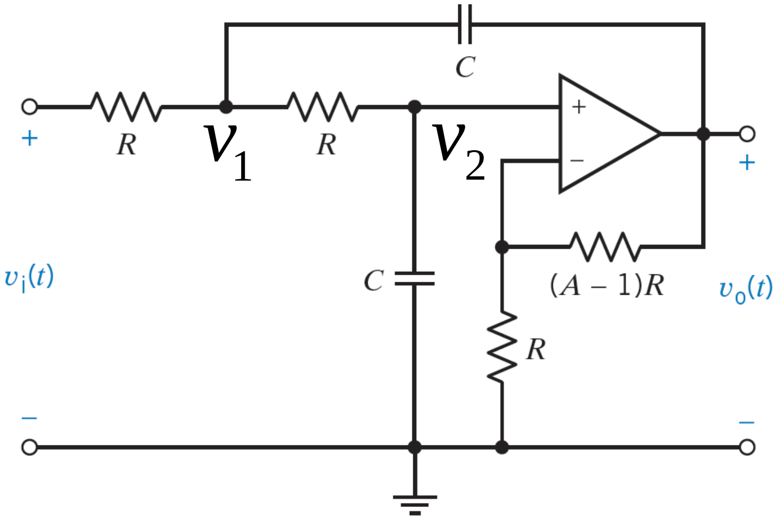
$$= \frac{R_f}{R_i} \frac{2\omega_1}{\omega_1 + \omega_2}$$

Sallen-Key Filters

FILTER TYPE	CIRCUIT	DESIGN EQUATIONS
Low-pass		$\omega_0 = \frac{1}{RC}$ $Q = \frac{1}{3 - A}$ $k = A$

(continued)

Sallen-Key Low-Pass Filters



$$V_2 = \frac{R}{R+(A-1)R} V_o = \frac{1}{A} V_o$$

$$V_2 = \frac{\frac{1}{sC}}{R+\frac{1}{sC}} V_1 = \frac{1}{sRC+1} V_1 = \frac{1}{A} V_o \Rightarrow V_1 = \frac{sRC+1}{A} V_o$$

$$\frac{V_i - V_1}{R} + \frac{V_o - V_1}{\frac{1}{sC}} + \frac{V_2 - V_1}{R} = \frac{V_i - V_1}{R} + sC\left(V_o - V_1\right) + \frac{V_2 - V_1}{R} = 0$$

$$V_i - V_1 + sRC\left(V_o - V_1\right) + V_2 - V_1 = 0$$

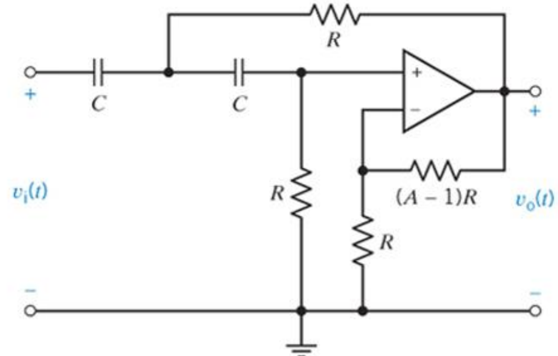
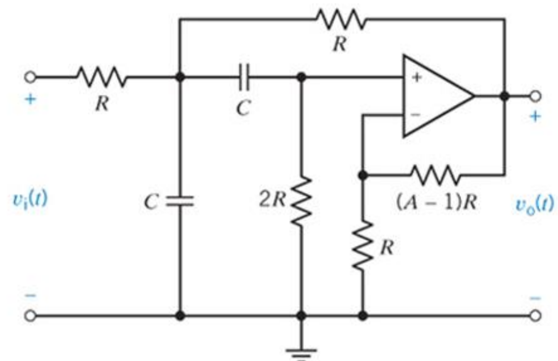
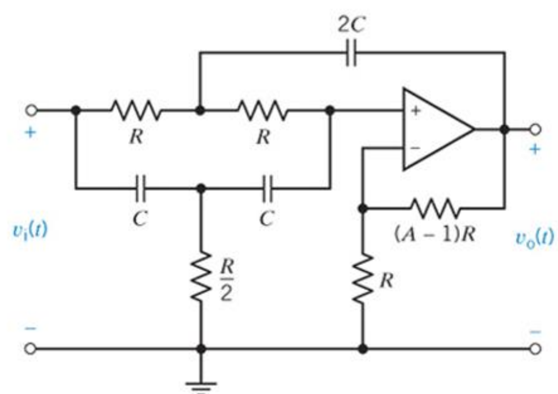
$$V_o = \frac{\frac{A}{(RC)^2}}{s^2 + \frac{3-A}{RC}s + \left(\frac{1}{RC}\right)^2} V_i$$

$$H_L(s) = \frac{k\omega_0^{\,2}}{s^2 + \frac{\omega_0}{Q}s + \omega_0^{\,2}}$$

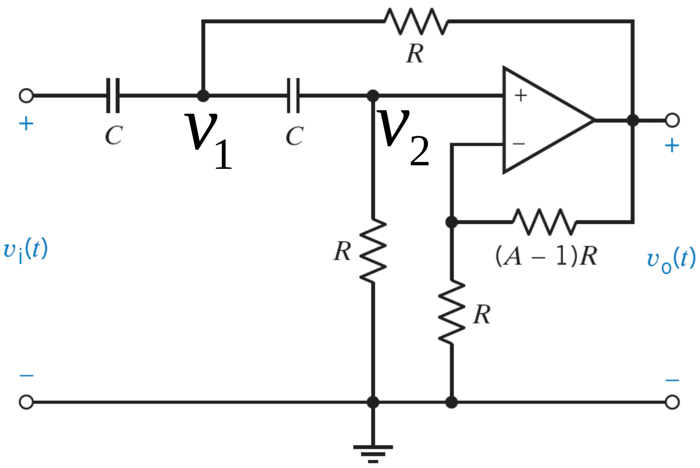
$$\omega_0 = \frac{1}{RC}$$

$$Q = \frac{1}{3-A}$$

$$k = A$$

FILTER TYPE	CIRCUIT	DESIGN EQUATIONS
High-pass		$\omega_0 = \frac{1}{RC}$ $Q = \frac{1}{3 - A}$ $k = A$
Band-pass		$\omega_0 = \frac{1}{RC}$ $Q = \frac{1}{3 - A}$ $k = AQ$
Band-stop (notch)		$\omega_0 = \frac{1}{RC}$ $Q = \frac{1}{4 - 2A}$ $k = A$

Sallen-Key High-Pass Filter



$$V_2 = \frac{R}{R+(A-1)R}V_o = \frac{1}{A}V_o$$

$$V_2 = \frac{R}{R+\frac{1}{sC}}V_1 = \frac{sRC}{sRC+1}V_1 = \frac{1}{A}V_o \Rightarrow V_1 = \frac{sRC+1}{sRCA}V_o$$

$$\frac{V_i - V_1}{\frac{1}{sC}} + \frac{V_o - V_1}{R} + \frac{V_2 - V_1}{\frac{1}{sC}} = sC\left(V_i - V_1\right) + \frac{V_o - V_1}{R} + sC\left(V_2 - V_1\right) = 0$$

$$sRC\left(V_i - V_1\right) + V_o - V_1 + sRC\left(V_2 - V_1\right) = 0$$

$$V_o = \frac{As^2}{s^2 + \frac{3-A}{RC}s + \left(\frac{1}{RC}\right)^2}V_i$$

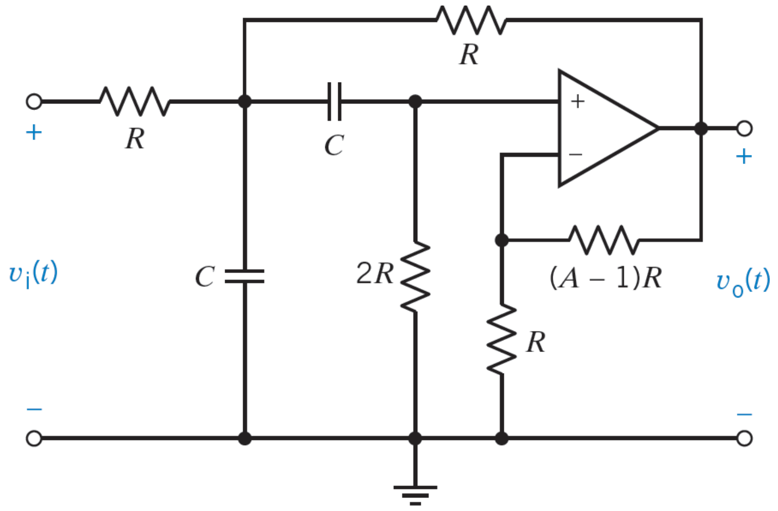
$$H(s) = \frac{ks^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$\omega_0 = \frac{1}{RC}$$

$$Q = \frac{1}{3-A}$$

$$k = A$$

Sallen-Key Band-Pass Filter



$$V_o = \frac{\frac{A}{RC}s}{s^2 + \frac{3-A}{RC}s + \left(\frac{1}{RC}\right)^2} V_i$$

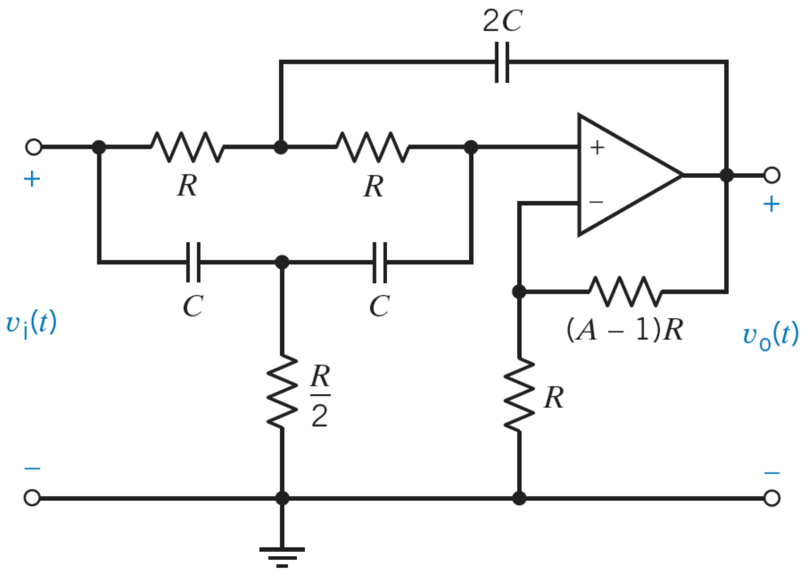
$$H(s) = \frac{k\frac{\omega_0}{Q}s}{s^2 + \frac{\omega_0}{Q}s + {\omega_0}^2}$$

$$\omega_0 = \frac{1}{RC}$$

$$Q = \frac{1}{3 - A}$$

$$k = AQ$$

Sallen-Key Band-Reject Filter



$$V_o = \frac{A\left(s^2 + \left(\frac{1}{RC}\right)^2\right)}{s^2 + \frac{4-2A}{RC}s + \left(\frac{1}{RC}\right)^2} V_i$$

$$H(s) = \frac{k(s^2 + \omega_0{}^2)}{s^2 + \frac{\omega_0}{Q}s + \omega_0{}^2}$$

$$\omega_0 = \frac{1}{RC}$$

$$Q = \frac{1}{4 - 2A}$$

$$k = A$$

Try it yourself in WileyPLUS

EXAMPLE 16.4-2 Sallen-Key Band-Pass Filter

Design a second-order Sallen-Key band-pass filter with a center frequency of 500 hertz and a bandwidth of 100 hertz.

Solution

The transfer function of the second-order band-pass filter is

$$H(s) = \frac{k\dfrac{\omega_0}{Q}s}{s^2 + \dfrac{\omega_0}{Q}s + {\omega_0}^2}$$

The corresponding network function is

$$\mathbf{H}(\omega) = \frac{jk\dfrac{\omega_0}{Q}\omega}{{\omega_0}^2 - \omega^2 + j\dfrac{\omega_0}{Q}\omega}$$

Dividing numerator and denominator by $j\dfrac{\omega_0}{Q}\omega$ gives

$$\mathbf{H}(\omega) = \frac{k}{1 + jQ\left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)}$$

We have seen network functions like this one earlier, when we discussed resonant circuits in Chapter 13. The gain $|\mathbf{H}(\omega)|$ will be maximum at the corner frequency ω_0. In the case of this band-pass transfer function, ω_0 is also called the center frequency and the resonant frequency. The gain at the center frequency will be

$$|\mathbf{H}(\omega_0)| = k$$

Two frequencies, ω_1 and ω_2, are identified by the property

$$|\mathbf{H}(\omega_1)| = |\mathbf{H}(\omega_2)| = \frac{k}{\sqrt{2}}$$

These frequencies are called the *half-power frequencies* or the 3 *dB frequencies*. The half-power frequencies are given by

$$\omega_1 = -\frac{\omega_0}{2Q} + \sqrt{\left(\frac{\omega_0}{2Q}\right)^2 + \omega_0{}^2} \quad \text{and} \quad \omega_2 = \frac{\omega_0}{2Q} + \sqrt{\left(\frac{\omega_0}{2Q}\right)^2 + \omega_0{}^2}$$

The bandwidth of the filter is calculated from the half-power frequencies

$$BW = \omega_2 - \omega_1 = \frac{\omega_0}{Q}$$

The Sallen-Key band-pass filter is shown in the third row of Table 16.4-2. Our specifications require that

$$\omega_0 = 2\pi \cdot 500 = 3142 \text{ rad/s}$$

and

$$Q = \frac{\omega_0}{BW} = 5$$

From Table 16.4-2, the design equations for the Sallen-Key band-pass filter are

$$\frac{1}{RC} = \omega_0 = 3142$$

and

$$A = 3 - \frac{1}{Q} = 2.8$$

Pick $C = 0.1\ \mu\text{F}$. Then

$$R = \frac{1}{C\omega_0} = 3183\ \Omega$$

Because $k = AQ$, the gain of this band-pass filter at the center frequency is 14. Also, one of the resistances is given by

$$(A - 1)R = 5729\ \Omega$$

The Sallen-Key band-pass filter is shown in Figure 16.4-2.

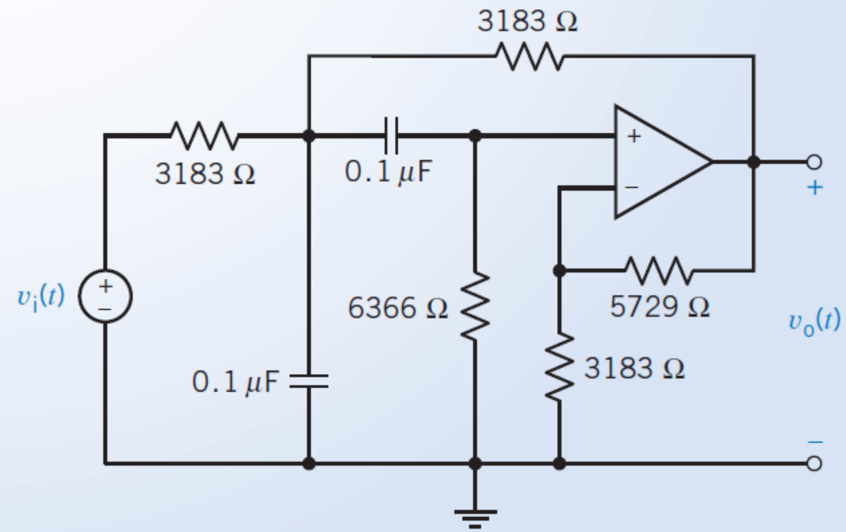


FIGURE 16.4-2 A Sallen-Key band-pass filter.

Example 16.4-3 Sallen-Key Band-Stop Filter

Design a second-order band-stop filter with a center frequency of 1000 rad/s and a bandwidth of 100 rad/s.

Solution

The transfer function of the second-order band-stop filter is

$$H(s) = \frac{k(s^2 + {\omega_0}^2)}{s^2 + \dfrac{\omega_0}{Q}s + {\omega_0}^2}$$

Notice that the transfer functions of the second-order band-pass and band-stop filters are related by

$$\frac{k(s^2 + {\omega_0}^2)}{s^2 + \dfrac{\omega_0}{Q}s + {\omega_0}^2} = k - \frac{k\dfrac{\omega_0}{Q}s}{s^2 + \dfrac{\omega_0}{Q}s + {\omega_0}^2}$$

The network function of the band-stop filter is

$$\mathbf{H}(\omega) = \frac{k({\omega_0}^2 - \omega^2)}{{\omega_0}^2 - \omega^2 + j\dfrac{\omega_0}{Q}\omega}$$

When $\omega \ll \omega_0$ or $\omega \gg \omega_0$, the gain is $|\mathbf{H}(\omega)| = k$. At $\omega = \omega_0$, the gain is zero. The half-power frequencies ω_1 and ω_2 are identified by the property

$$|\mathbf{H}(\omega_1)| = |\mathbf{H}(\omega_2)| = \frac{k}{\sqrt{2}}$$

The bandwidth of the filter is given by

$$BW = \omega_2 - \omega_1 = \frac{\omega_0}{Q}$$

The Sallen-Key band-stop filter is shown in the last row of Table 16.4-2. Our specifications require that $\omega_0 = 1000$ rad/s and

$$Q = \frac{\omega_0}{BW} = 10$$

Table 16.4-2 indicates that the design equations for the Sallen-Key band-stop filter are

$$\frac{1}{RC} = \omega_0 = 1000$$

and

$$A = 2 - \frac{1}{2Q} = 1.95$$

Pick $C = 0.1\ \mu\text{F}$. Then

$$R = \frac{1}{C\omega_0} = 10\ \text{k}\Omega$$

The Sallen-Key band-stop filter is shown in Figure 16.4-3.

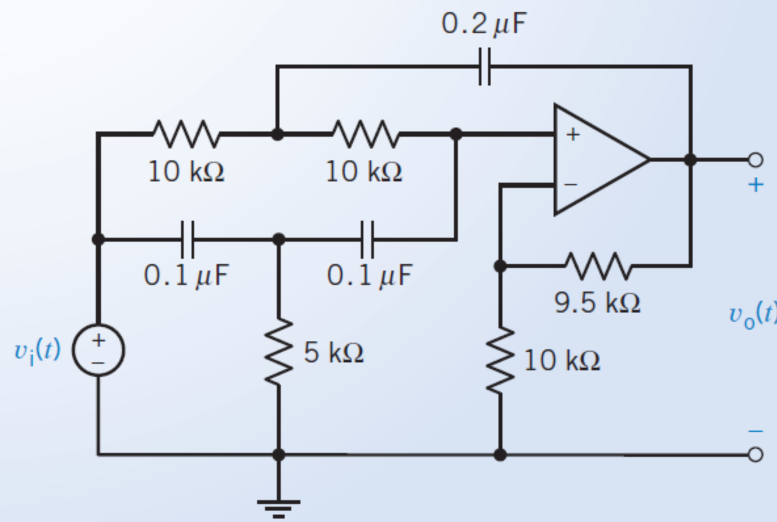


FIGURE 16.4-3 A Sallen-Key band-stop filter.