RC & RL Circuits

Differential Equation

- Algebraic Equation

$$x + x = 1$$

$$2x = 1$$

$$x = 0.5$$

- Differential Equation

$$\frac{dx}{dt} + x = \dot{x} + x = 0$$

$$\dot{x} = -x$$

- Candidates

$$x = t^2, \dot{x} = 2t$$

$$x = \cos t, \dot{x} = -\sin t$$

$$x = Ke^{\lambda t}, \dot{x} = K\lambda e^{\lambda t}$$

Solution

$$\dot{x} = -x$$

$$\dot{x} = K\lambda e^{\lambda t} = -x = -Ke^{\lambda t}$$

$$K\lambda e^{\lambda t} + Ke^{\lambda t} = K(\lambda + 1)e^{\lambda t}$$

$$\lambda = -1$$

$$x = Ke^{-t}$$

$$x(0) = K$$

$$x(t) = x(0)e^{-t}$$

Another Example

$$2\dot{x} + 3x = 0$$

$$x = Ke^{\lambda t}, \dot{x} = K\lambda e^{\lambda t}$$

$$2K\lambda e^{\lambda t} + 3Ke^{\lambda t} = 0$$

$$K(2\lambda + 3)e^{\lambda t} = 0$$

$$\lambda = -1.5$$

$$x = Ke^{-1.5t}$$

$$x(0) = K$$

Characteristic Equation

$$a\dot{x} + bx = 0$$

$$a\lambda + b = 0$$

$$\lambda = -\frac{b}{a} \Rightarrow e^{\lambda t}$$

Non-zero input

$$\dot{x} + x = 1$$

$$x(t) = K_1 e^{\lambda t} + K_2$$

$$\dot{x} + x = K_1 \lambda e^{\lambda t} + K_1 e^{\lambda t} + K_2 = 1$$

$$K_1(\lambda + 1)e^{\lambda t} + K_2 = 1$$

$$\lambda = -1, K_2 = 1$$

$$x(t) = K_1 e^{-t} + 1$$

$$\lim_{t \to \infty} x(t) = 1$$

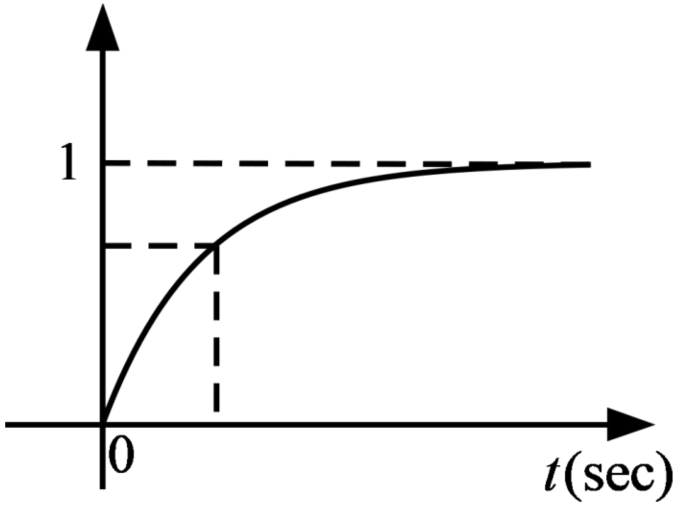
$$x(0) = K_1 + 1$$

$$K_1 = x(0) - 1$$

$$x(t) = \left(x(0) - 1\right)e^{-t} + 1$$

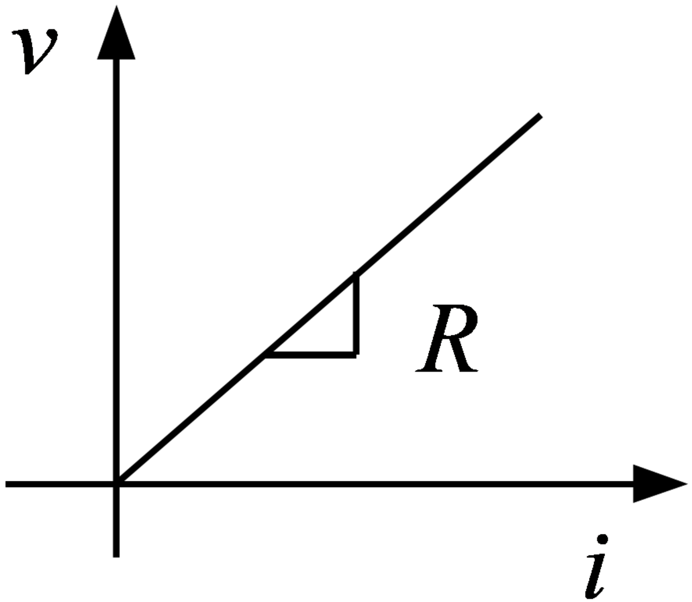
$$\text{if} : x(0) = 0$$

$$x(t) = -e^{-t} + 1$$

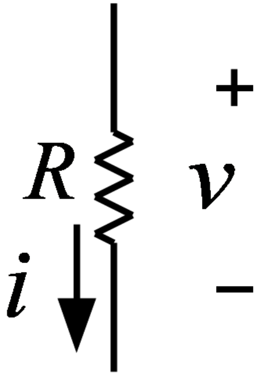


- Complete response = natural response + forced response
- Natural response(homogeneous solution), transient response
- Forced response(particular solution), steady state response

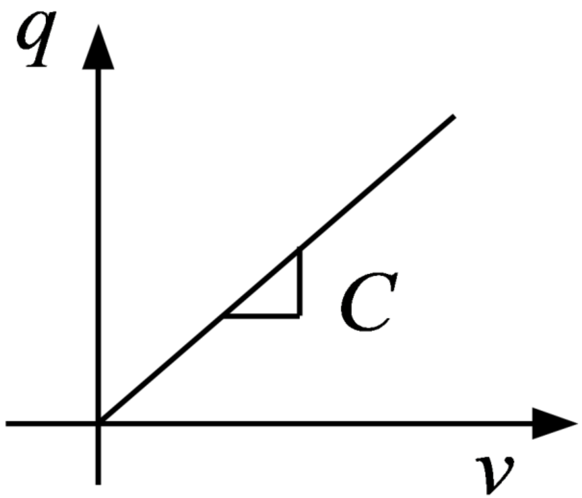
Resistor

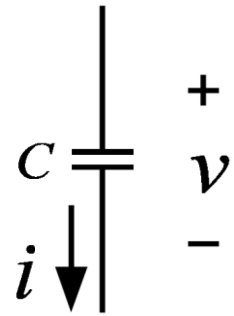


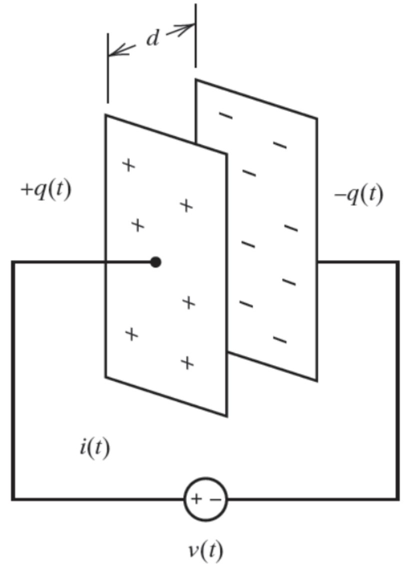
$$v = Ri$$



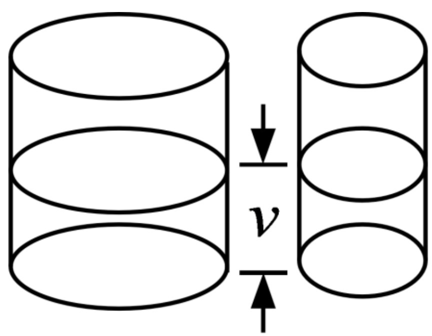
Capacitor







Charge: $q = Cv$

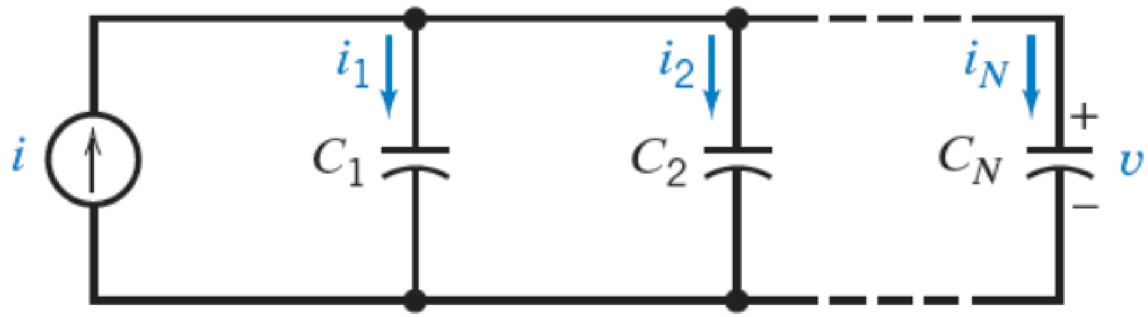


$$C_1 > C_2$$

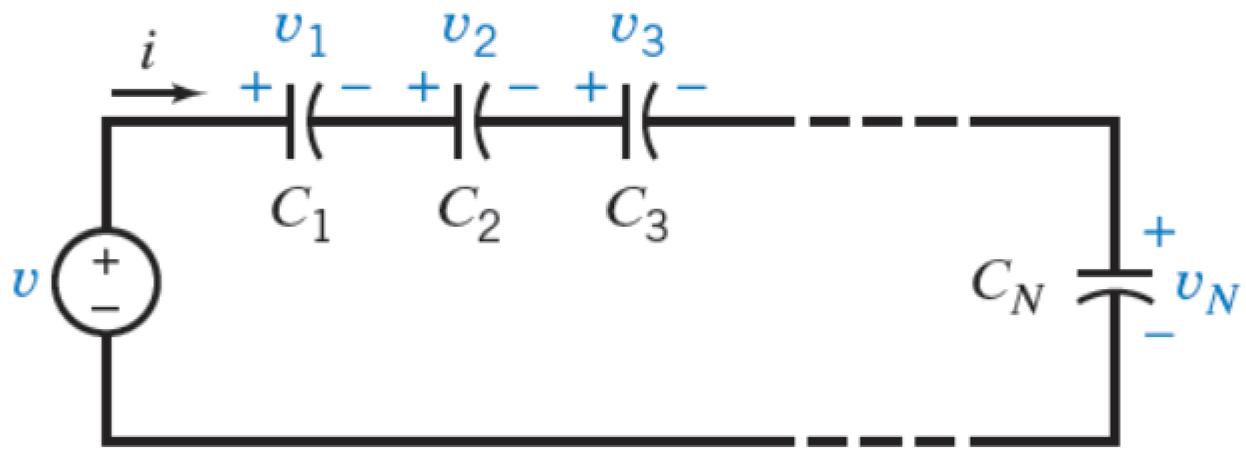
$$i = \frac{dq}{dt} = C\frac{dv}{dt}$$

$$q_1 = C_1 v > q_2 = C_2 v$$

Series and Parallel Capacitors

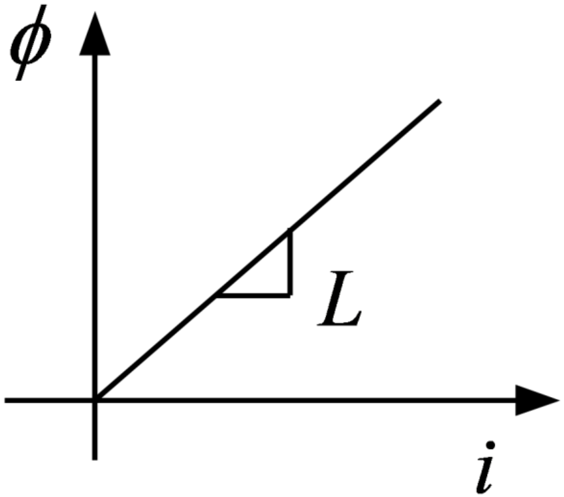


$$C_{\text{p}} = C_1 + C_2 + C_3 + \cdots + C_N = \sum_{n=1}^{N} C_n$$

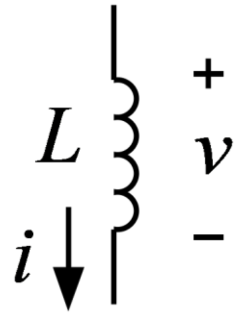


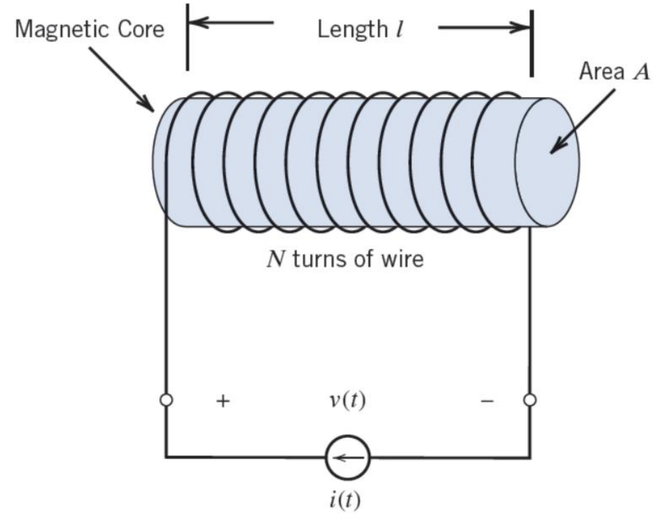
$$\frac{1}{C_s} = \sum_{n=1}^{N} \frac{1}{C_n}$$

Inductor



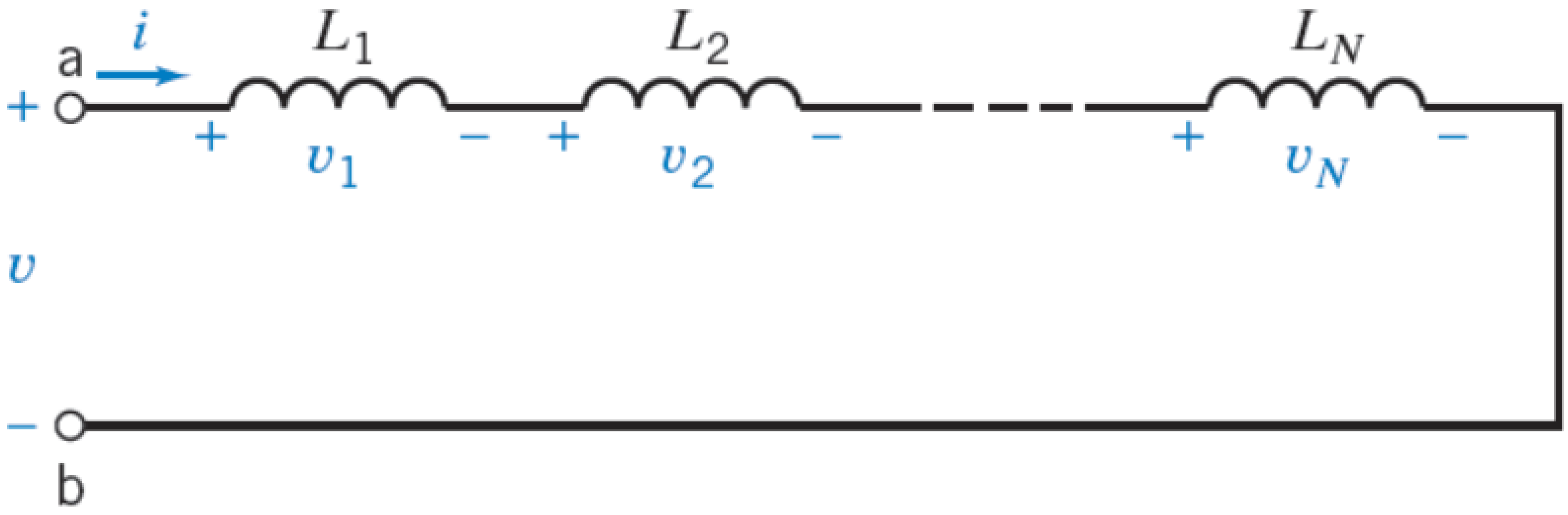
$$\phi = Li$$



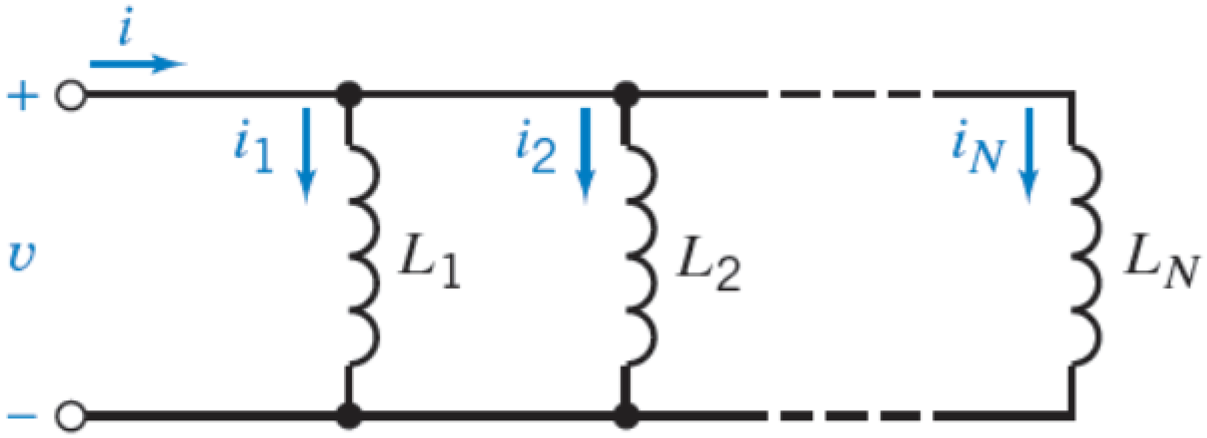


$$v = \frac{d\phi}{dt} = L\frac{di}{dt}$$

Series and Parallel Inductors

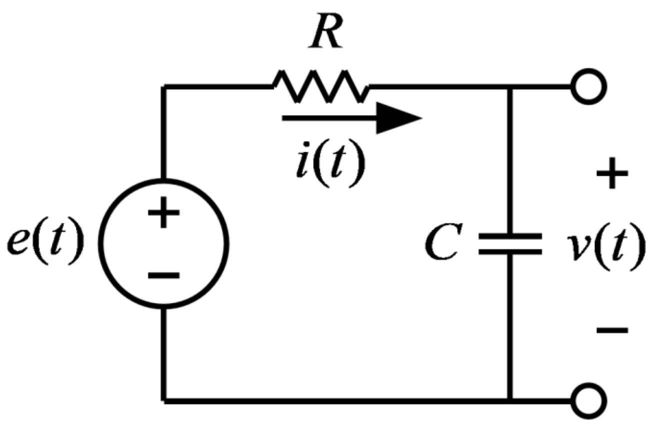


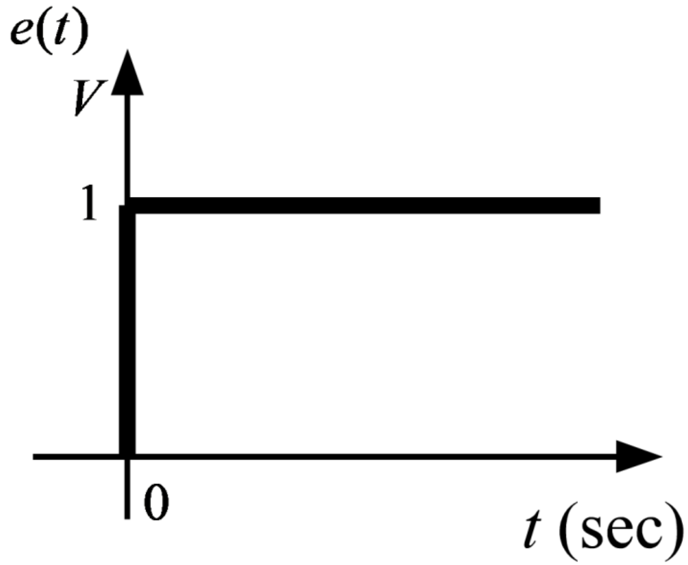
$$L_s = \sum_{n=1}^{N} L_n$$



$$\frac{1}{L_p} = \sum_{n=1}^{N} \frac{1}{L_n}$$

RC Circuit





$$KVL:$$

$$Ri + v = e, \quad i = C\frac{dv}{dt}$$

$$RC\frac{dv}{dt} + v = e, \quad v(0) = 0$$

- Complete response = natural response + forced response
- Natural response(homogeneous solution):

$$RC\frac{dv}{dt}+v=0$$

$$RCs+1=0 \Rightarrow s=-\frac{1}{RC}$$

$$v(t)=Ke^{-\frac{t}{RC}}$$

- Forced response(particular solution), steady state:

$$v=V_{ss}$$

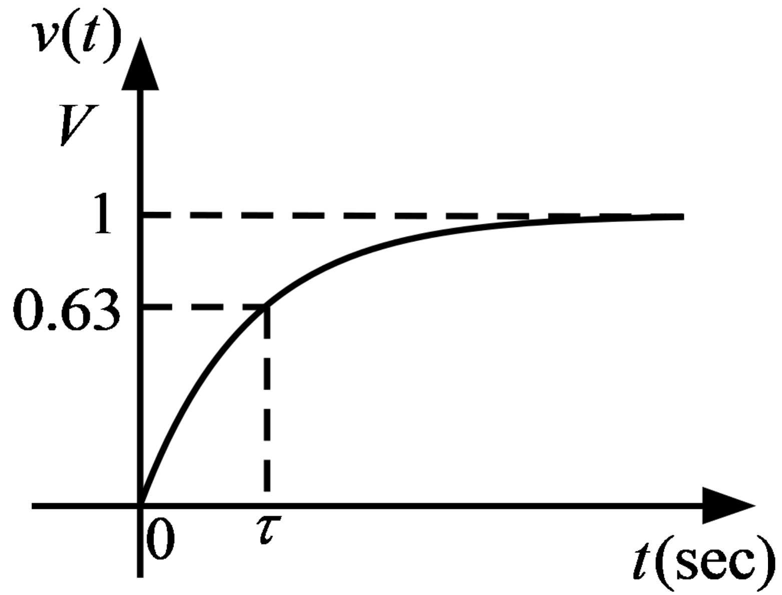
$$RC\frac{dV_{ss}}{dt}+V_{ss}=1 \Rightarrow V_{ss}=1$$

- Complete response = natural response + forced response

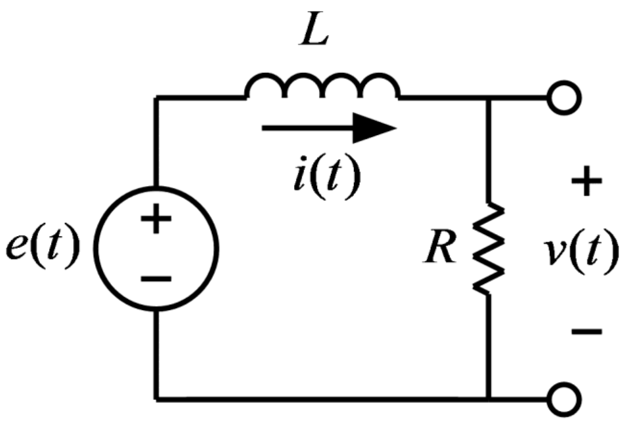
$$v(t) = Ke^{-\frac{t}{RC}} + 1$$

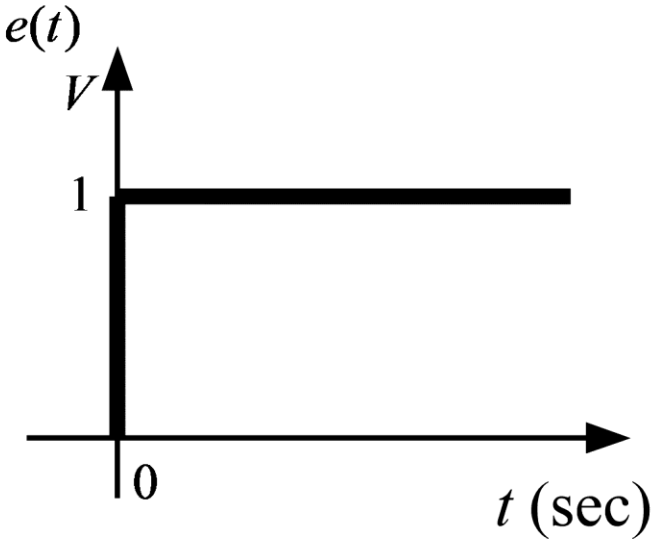
$$v(0) = K + 1 = 0 \Rightarrow K = -1$$

$$v(t) = -e^{-\frac{t}{RC}} + 1$$



RL Circuit





$$e(t) = L\frac{di}{dt} + v, i = \frac{v}{R}$$

$$\frac{L}{R}\frac{dv}{dt} + v = e(t), v(0) = 0$$

- Complete response = natural response + forced response
- Natural response(homogeneous solution):

$$\frac{L}{R}\frac{dv}{dt}+v=0$$

$$\frac{L}{R}s+1=0 \Rightarrow s=-\frac{L}{R}$$

$$v(t)=Ke^{-\frac{L}{R}t}$$

- Forced response(particular solution), steady state:

$$v=V_{ss}$$

$$\frac{L}{R}\frac{dV_{ss}}{dt}+V_{ss}=1 \Rightarrow V_{ss}=1$$

- Complete response = natural response + forced response

$$v(t) = Ke^{-\frac{L}{R}t} + 1$$

$$v(0) = K + 1 = 0 \Rightarrow K = -1$$

$$v(t) = -e^{-\frac{L}{R}t} + 1$$