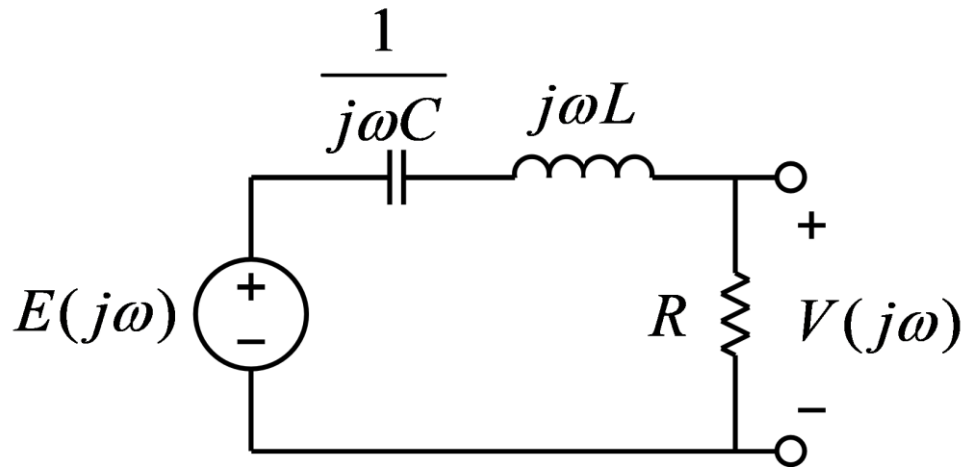

Resonant Circuits

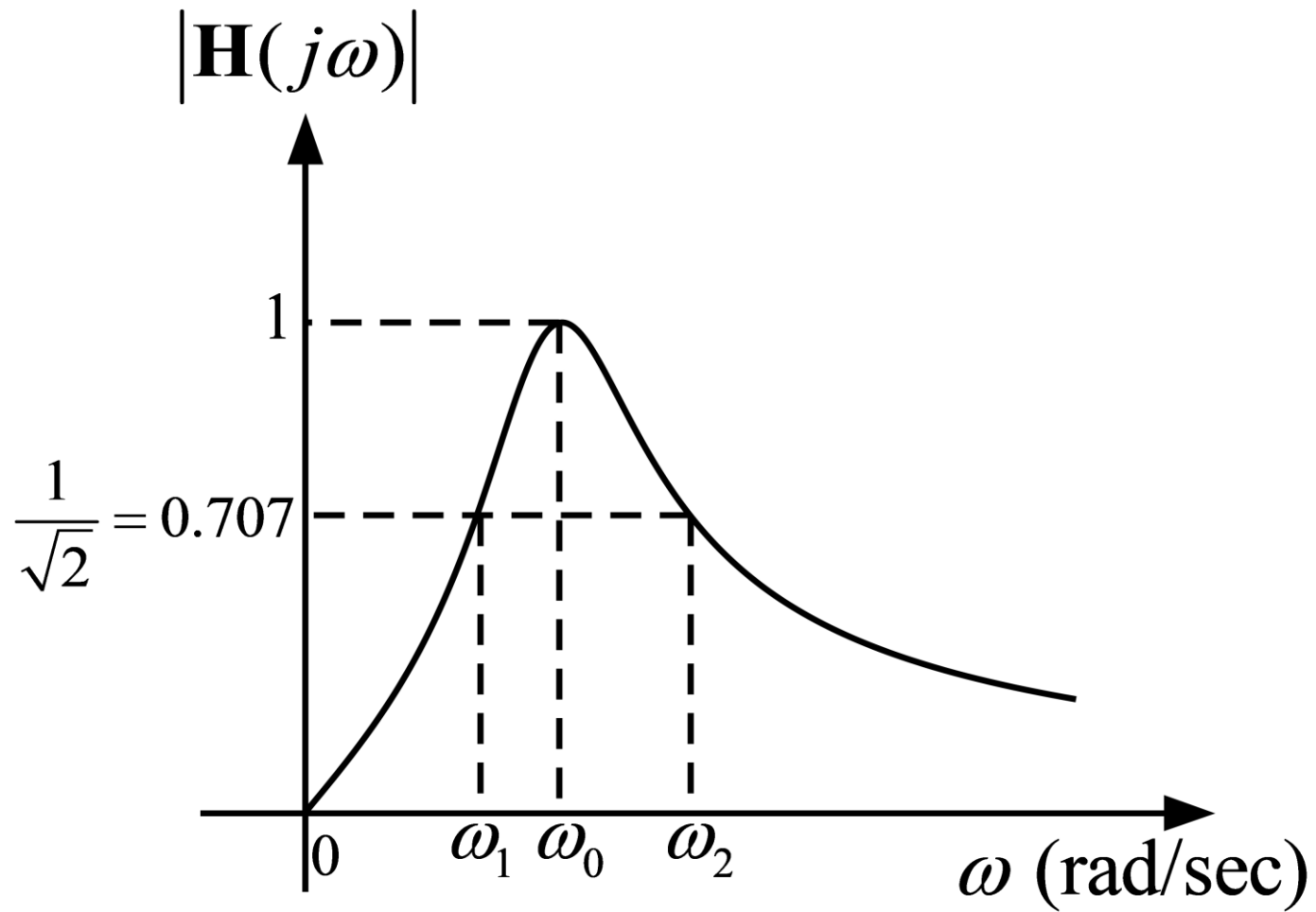
RLC Resonant Circuits

Series Resonant Circuits



$$V(j\omega) = \frac{R}{j\omega L + R + \frac{1}{j\omega C}} E(j\omega) = \frac{j\omega RC}{(j\omega)^2 LC + (j\omega)RC + 1} E(j\omega)$$

$$H(j\omega) = \frac{j\omega RC}{(j\omega)^2 LC + (j\omega)RC + 1} = \frac{j\omega \frac{R}{L}}{(j\omega)^2 + (j\omega) \frac{R}{L} + \frac{1}{LC}}$$



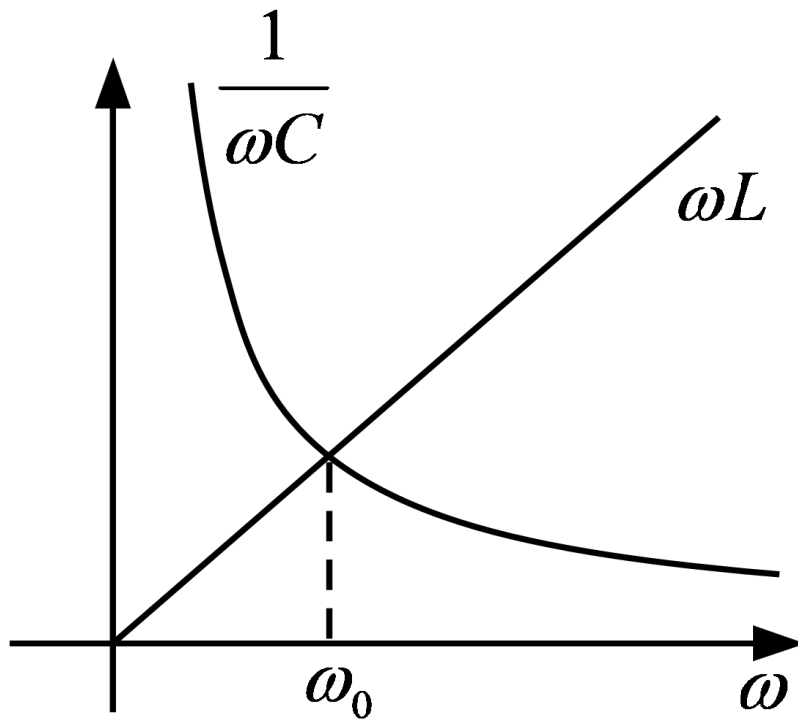
$$Z(j\omega) = j\omega L + R + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$|Z(j\omega)| = \left| R + j\left(\omega L - \frac{1}{\omega C}\right) \right| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$V(j\omega) = \frac{R}{Z(j\omega)} E(j\omega), G(j\omega) = \frac{V(j\omega)}{E(j\omega)} = \frac{R}{Z(j\omega)}$$

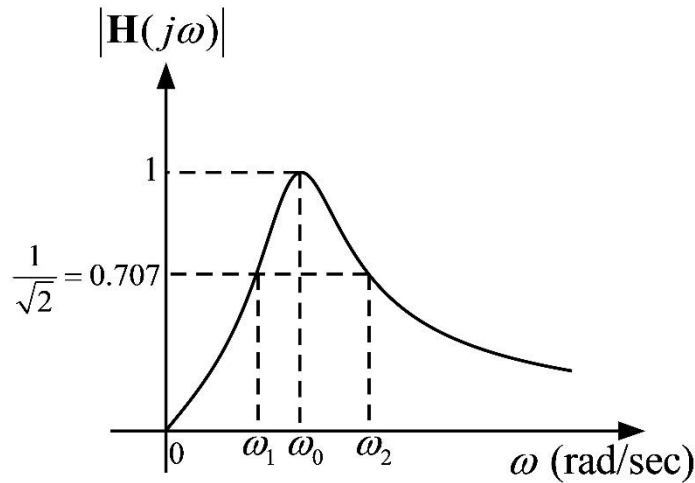
$$|H(j\omega)| = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\omega L = \frac{1}{\omega C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} : |H(j\omega)| = 1$$



$$\omega L = \frac{1}{\omega C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} : |G(j\omega)| = 1$$

Band Width, Quality Factor



$$B = \omega_2 - \omega_1$$

$$H(j\omega) = \frac{(j\omega) \left(\frac{\omega_0}{Q} \right)}{(j\omega)^2 + (j\omega) \left(\frac{\omega_0}{Q} \right) + \omega_0^2}$$

$$H(j\omega) = \frac{j\omega \frac{R}{L}}{(j\omega)^2 + (j\omega) \frac{R}{L} + \frac{1}{LC}}$$

$$Q = \frac{L\omega_0}{R} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

$$Q = \frac{\omega_0}{B} = \frac{\omega_0}{\omega_2 - \omega_1}$$

$$(j\omega)^2 + 2\zeta\omega_0(j\omega) + \omega_0^2 = (j\omega)^2 + (j\omega) \left(\frac{\omega_0}{Q} \right) + \omega_0^2$$

$$2\zeta\omega_0 = \frac{\omega_0}{Q} \Rightarrow Q = \frac{1}{2\zeta}$$

Impedance

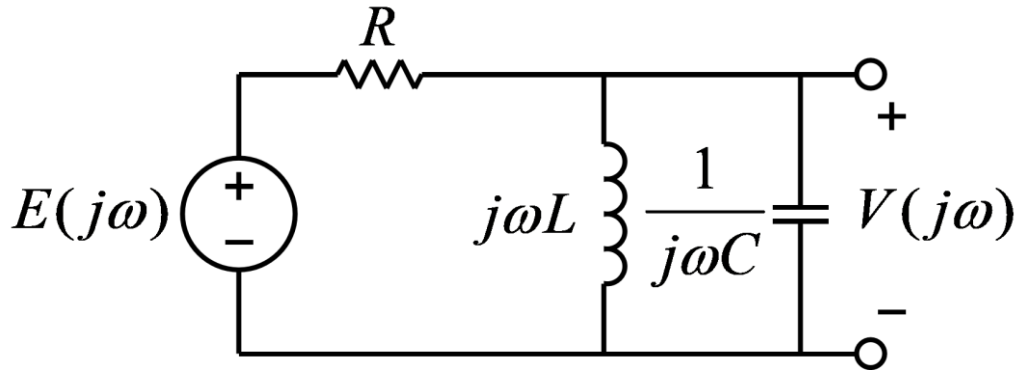
$$Z(j\omega) = j\omega L + R + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$= R\left(1 + j\frac{1}{R}\left(\omega L - \frac{1}{\omega C}\right)\right)$$

$$Z(j\omega) = R\left(1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right)$$

$$Q\frac{1}{\omega_0} = \frac{L}{R}, Q\omega_0 = \frac{1}{RC} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}, Q = \frac{L\omega_0}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Parallel Resonant Circuits

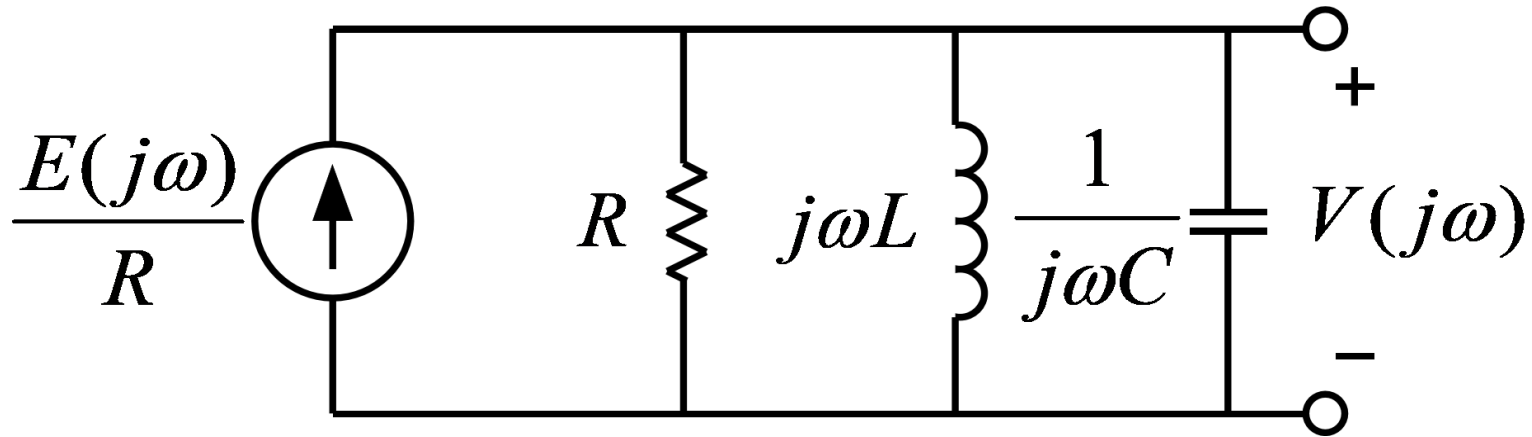


$$j\omega L \parallel \frac{1}{j\omega C} = \frac{\frac{j\omega L}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega L}{(j\omega)^2 LC + 1}$$

$$V(j\omega) = \frac{\frac{j\omega L}{(j\omega)^2 LC + 1}}{R + \frac{j\omega L}{(j\omega)^2 LC + 1}} E(j\omega) = \frac{j\omega L}{(j\omega)^2 LCR + (j\omega)L + R} E(j\omega)$$

$$H(j\omega) = \frac{j\omega \left(\frac{L}{R} \right)}{(j\omega)^2 LC + (j\omega) \left(\frac{L}{R} \right) + 1} = \frac{j\omega \left(\frac{1}{RC} \right)}{(j\omega)^2 + (j\omega) \left(\frac{1}{RC} \right) + \frac{1}{LC}}$$

Admittance



$$Y(j\omega) = \frac{1}{R} + j\omega C + \frac{1}{j\omega L} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

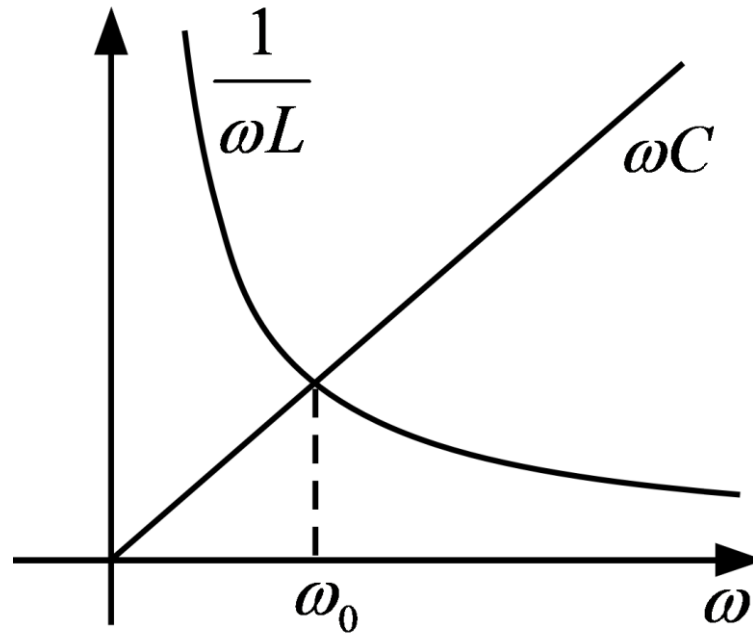
$$Y(j\omega) = \frac{1}{R} + j\omega C + \frac{1}{j\omega L} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

$$|Y(j\omega)| = \left| \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right) \right| = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

$$V(j\omega) = \frac{1}{Y(j\omega)} \frac{E(j\omega)}{R}$$

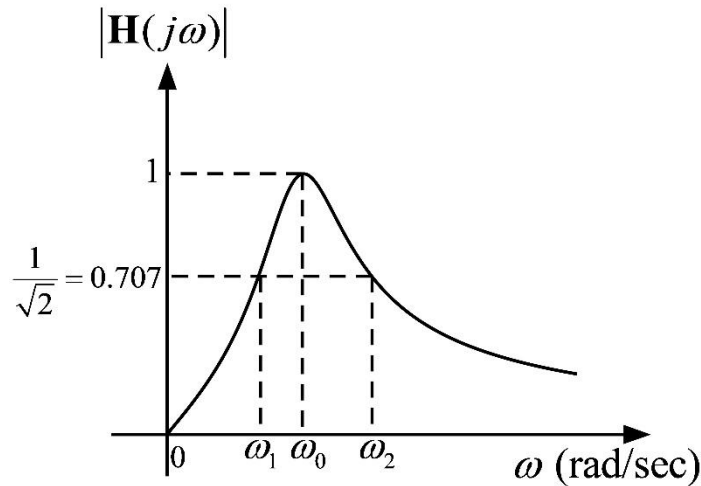
$$H(j\omega) = \frac{V(j\omega)}{E(j\omega)} = \frac{1}{R} \frac{1}{Y(j\omega)}$$

$$|H(j\omega)| = \frac{1}{R} \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$



$$\omega L = \frac{1}{\omega C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} : |G(j\omega)| = 1$$

Band Width, Quality Factor



$$B = \omega_2 - \omega_1$$

$$H(j\omega) = \frac{(j\omega)\left(\frac{\omega_0}{Q}\right)}{(j\omega)^2 + (j\omega)\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

$$H(j\omega) = \frac{j\omega\left(\frac{1}{RC}\right)}{(j\omega)^2 + (j\omega)\left(\frac{1}{RC}\right) + \frac{1}{LC}}$$

$$Q = RC\omega_0 = R\sqrt{\frac{C}{L}}$$

$$Q = \frac{\omega_0}{B} = \frac{\omega_0}{\omega_2 - \omega_1}$$

Admittance

$$Y(j\omega) = \frac{1}{R} + j\omega C + \frac{1}{j\omega L} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

$$= \frac{1}{R} \left(1 + jR \left(\omega C - \frac{1}{\omega L} \right) \right)$$

$$Y(j\omega) = \frac{1}{R} \left(1 + jQ \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) \right)$$

$$Q \frac{1}{\omega_0} = RC, Q\omega_0 = \frac{R}{L} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}, Q = RC\omega_0 = R\sqrt{\frac{C}{L}}$$